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ROYAL ASTRONOMICAL SOCIETY.

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WILLIAM HUGGINS, ESQ., F.R.S., President, in the Chair.

William Morris Beaufort, Esq., 18 Piccadilly, and Athenæum Club,

was balloted for and duly elected a Fellow of the Society.

Extracts from Letters received by the Astronomer Royal, relating to the Search for the supposed Intra-Mercurial Planet.

The post has now brought letters in reply to some of the telegrams which were addressed to distant stations requesting a search for the supposed Intra-Mercurial Planet. The following is the substance of the information conveyed.

From R. N. Pogson, Esq., Government Observatory, Madras.

"Eye-observations were made at nearly every ten minutes on March 21, 22, 23, and fourteen photographs were taken on March 22. *Vulcan* was not seen."

From Charles Todd, Esq., General Post Office, Adelaide.

"The weather was excessively bad, with clouds and thunderstorm; $3\frac{1}{2}$ inches of rain fell in Adelaide, and 5 or 6 inches in the neighbourhood. Observations were attempted, but *Vulcan* was not seen."

From R. L. J. Ellery, Esq., The Observatory, Melbourne.

"The weather was fine, the Sun was observed almost uninterruptedly, and numerous photographs were taken. No appearance of *Vulcan*."

Royal Observatory, Greenwich,
1877, *April* 7.

Sun Spot Drawings of the Rev. Frederick Howlett.

In the short account given in the May number of the *Monthly Notices* for 1876 of the volumes of Sun Spot Drawings which have been presented to the Library of the Astronomical Society by the Rev. F. Howlett, it should have been mentioned that a large number of the drawings of the Sun's disk are marked with a small line and figures, indicating the distance of the west point of the Sun's limb from the vertex or highest point of the Sun above the horizon at the time of observation.

The image of the Sun was projected on a screen on which was a graduated circle with 0° at the vertex, 90° being to the right hand, 180° at the bottom, and 270° to the left hand. A small spot, or some easily recognised detail of an irregular large spot or large group, was brought to the centre of the graduated circle, which was marked by a small pin-hole in the cardboard screen. The telescope was then left in position, and as the Sun's image was carried off the screen by the Earth's motion, the angular distance from the vertex at which the spot or detail which had been brought to the centre passed across the circumference of the graduated circle was noted and entered upon the drawing. The position of a small line which has been marked upon the edge of the drawings of the Sun's disk depends on the assumption, that the highest point upon the page of the book accurately corresponded with the highest point of the Sun's image as seen by projection on the screen.

The horizontal and vertical co-ordinates of spots given in some of the drawings were made with an eye-piece in the focus of which was a thin slip of semi-transparent mother-of-pearl, graduated into divisions $\frac{1}{100}$th of an inch apart, every 5th and 10th gradation being made more conspicuous for the purpose of easily counting them. At right angles to the scale, and nearly bisecting the field of view, was stretched a fine wire, which was first placed vertically, so as to bisect the image of the Sun; the mother-of-pearl scale was now brought up to the centre of the spot, and the number of gradations to the right or left of the wire were counted; the eye-piece was then turned so that the wire when in a horizontal position bisected the Sun's image, and the mother-of-pearl scale brought again to the spot, and the number of gradations above or below the wire counted as before. It will be seen that these distances do not correspond to the horizontal and vertical ordinates of Pastorff's drawings, in which the distances measured horizontally and vertically were measured from the limbs of the Sun.

Note on the alleged Fall of an Aërolite at Supino, Italy.

By Padre Secchi.

(*From a Letter to E. Dunkin, Esq., F.R.S.*)

In the *Monthly Notices*, vol. xxxvii., No. 4, February 1877, page 205, I find indicated a fall of a meteor on September 4, 1875, 4 P.M. local time, at Supino, near Rome, in the province of Frosinone. As I am well acquainted with the particulars of this event, I can assure you that this is an equivocation, and a very great mystification. I have had the stone in my hand, and there is not the least appearance of the classical crust or varnish of aërolites, nor of the composition of the meteoric stones I have seen and so attentively examined at the British Museum and at the Jardin des Plantes.

It is not difficult to understand from the circumstances attending the fall how this equivocation has originated. It was a very stormy evening, and an electrical explosion took place, which traversed a gallery in the house of the parish priest. A nail was pointed out against which the flame of the alleged aërolite struck. At the same time some noise of a fallen body was heard, and a stone not previously existing in that court was found; hence the conclusion that a meteor had fallen at Supino. A German assistant to a Professor at the University went immediately to Supino and gave fifty francs for a piece of the stone. The owner of the house thought that he had really received *from the heavens* a treasure to provide the means for a dotation to a nephew on his approaching marriage, and, as this was near, he asked fabulous sums for the remainder of the stone. Prof. De Rossi visited the place and was persuaded that the gallery could not have been traversed by the stone, but only by the lightning. No hole of any depth was found in the place where the stone fell. He brought it to Rome; and when I saw it we were convinced that it was only a piece of volcanic pumice, which is frequently found in that district, an extinct volcano being at no great distance.

The fall may be explained thus. In this country it is customary to place stones on the tiled roofs of the houses, to prevent the tiles from being blown away by the winds. A volcanic stone was probably on the roof, and the lightning struck against it as a more conducting material, and threw it into the court. This explanation, which is very simple, is very probable; the noise heard was not that of a meteor, but that of the thunder storm.

I have considered it my duty to inform the Society of the above facts, to avoid mystification.

Rome, April 19, 1877.

On an Elliptic Function Solution of Kepler's Problem.

By J. W. L. Glaisher, M.A., F.R.S.

§ 1. In a paper recently published by the German Astronomical Society,* Dr. Hugo Gyldén has applied the Theory of Elliptic Functions to the solution of Kepler's Problem in the following manner:—

Writing the equations in the form

$$\tan\tfrac{1}{2}f=\sqrt{\left(\frac{1+e}{1-e}\right)}\tan\tfrac{1}{2}\mathrm{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\mathrm{M}=\mathrm{E}-e\sin\mathrm{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)\dagger$$

(where $\mathrm{M}, = nt+c$, is the mean anomaly, E the eccentric anomaly, and f the true anomaly) Dr. Gyldén takes $\frac{1}{2}\mathrm{E}$ equal to the amplitude of an elliptic integral—viz. he puts

$$\tfrac{1}{2}\mathrm{E}=\mathrm{am}\,v,$$

the modulus k being given by the equation

$$k=\sqrt{\left(\frac{2e}{1+e}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

so that $k'=\sqrt{\left(\frac{1-e}{1+e}\right)}$, and thus the equations (1) and (2) become, on writing $\frac{1}{2}f=\mathrm{am}\,u$,

$$\mathrm{coam}\,u=90^\circ-\mathrm{am}\,v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\mathrm{M}=2\,\mathrm{am}\,v-e\sin(2\,\mathrm{am}\,v) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The quantities r, x, y are also readily expressible in terms of elliptic functions of u or v.

The equation (5) may be written

$$\mathrm{M}=2\,\mathrm{am}\,v-2e\,\mathrm{sn}\,v\,\mathrm{cn}\,v$$

$$=2\,\mathrm{am}\,v+\frac{2e}{k^2}\frac{d^2}{dv^2}(\mathrm{am}\,v) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and from the Theory of Elliptic Functions we know that

$$2\,\mathrm{am}\,v=2\eta+\frac{4q}{1+q^2}\sin 2\eta+\frac{1}{2}\frac{4q^2}{1+q^4}\sin 4\eta+\&\mathrm{c}. \quad . \quad . \quad (7)$$

* "Zur Auflösung des Kepler'schen Problemes." *Vierteljahrsschrift der Astronomischen Gesellschaft*, t. x. (1875), pp. 285-296.

† I have slightly altered the notation: Dr. Gyldén writes (2), $g=\epsilon-e\sin\epsilon$.

η being written in place of $\frac{\pi}{2K}v$, so that, taking account of the value of e given by (3), the equation (6) becomes

$$M = 2\eta - \left[\frac{4.1^2}{1+k'^2}\left(\frac{\pi}{2K}\right)^2 - 1\right]\frac{4q}{1+q^2}\sin 2\eta$$

$$-\frac{1}{2}\left[\frac{4.2^2}{1+k'^2}\left(\frac{\pi}{2K}\right)^2 - 1\right]\frac{4q^2}{1+q^4}\sin 4\eta$$

$$-\frac{1}{3}\left[\frac{4.3^2}{1+k'^2}\left(\frac{\pi}{2K}\right)^2 - 1\right]\frac{4q^3}{1+q^6}\sin 6\eta$$

$$- \&c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This equation is used to replace (2), viz.—given e we can calculate k from (3), then q from k, by known elliptic function formulæ, and $\log\frac{2K}{\pi}$ from q, by means of the equation

$$\frac{2K}{\pi} = (1 + 2q + 2q^4 + 2q^9 + \&c.)^2.$$

We thus obtain the numerical values of the coefficients of $\sin 2\eta$, $\sin 4\eta$, &c., in (8), and can determine by successive approximation the value of 2η, corresponding to a given value of M, and thence from (7), the value of 2 am v, that is, of E.

§ 2. Thus, in this method, the expression (2) is replaced by the two formulæ (7) and (8), which, though more complicated, are such that the convergence of the successive approximations is more rapid; in fact, the simplicity and slow convergence of (2) are exchanged for the greater complication and quicker convergence of (7) and (8). Of course, the question of whether the exchange is an advantageous one can only be definitively settled by careful numerical comparisons.

In what follows I propose to discuss the general nature of such an elliptic function solution of the problem as Dr. Gyldén proposes, and to examine, in particular, what value should be assigned to the modulus in order that the convergence of the approximations may be rendered as rapid as possible. It will be shown also that the calculation of the coefficients in (7) and (8) may be much facilitated; and some remarks upon the question of the practical utility of the method will be found in § 13.

§ 3. It is readily seen that the solution of the equation

$$x = a + n\phi(x),$$

where n is small, by successive approximation is equivalent to expanding x in powers of n by Lagrange's theorem, viz.:—

First approximation $x = a$;

Second „ $x = a + n\phi a$;

Third „ $x = a + n\phi\,(a + n\phi a)$

$= a + n\phi a + n^2\phi a\phi' a +$ &c.;

Fourth „ $x = a + n\phi\,(a + n\phi a + n^2\phi a\phi' a)$

$= a + n\phi a + n^2\phi a\phi' a + n^3\phi a\,(\phi' a)^2 + \frac{1}{2}n^3(\phi a)^2\,\phi'' a +$ &c.

$= a + n\phi a + \frac{n^2}{1.2}\frac{d}{da}(\phi a)^2 + \frac{n^3}{1.2.3}\frac{d^2}{da^2}(\phi a)^3 +$ &c.

so that the rth approximation differs from the truth by a quantity of the order n^r; and by the diminution of n the rapidity of the convergence of the approximation is very considerably increased.

The object in view is thus to render the coefficients of $\sin 2\eta$, $\sin 4\eta$, $\sin 6\eta$, &c., as small as possible, without unduly increasing the number of terms that must be included.

Resuming the equations (6) and (7), we have

$$2\,\mathrm{am}\,v = 2\eta + \frac{4q}{1+q^2}\sin 2\eta + \frac{1}{2}\frac{4q^2}{1+q^4}\sin 4\eta + \&c.$$

$$2\left(\frac{k\mathrm{K}}{\pi}\right)^2 \mathrm{sn}\,v\,\mathrm{cn}\,v = \frac{4q}{1+q^2}\sin 2\eta + 2\,\frac{4q^2}{1+q^4}\sin 4\eta + \&c.$$

so that, writing for brevity

$$\mathrm{R} = \left(\frac{\pi}{k\mathrm{K}}\right)^2,$$

we have

$$\begin{aligned}\mathrm{M} &= 2\,\mathrm{am}\,v - 2e\,\mathrm{sn}\,v\,\mathrm{cn}\,v\\ &= 2\eta - (\mathrm{Re}-1)\,\frac{4q}{1+q^2}\sin 2\eta\\ &\quad -\left(2\mathrm{Re}-\frac{1}{2}\right)\frac{4q^2}{1+q^4}\sin 4\eta\\ &\quad -\left(3\mathrm{Re}-\frac{1}{3}\right)\frac{4q^3}{1+q^6}\sin 6\eta\\ &\quad -\&c. \qquad\qquad (9)\end{aligned}$$

or, as we may write it,

$$\begin{aligned}\mathrm{M} &= 2\eta - (\mathrm{A}_1 e - \mathrm{B}_1)\sin 2\eta\\ &\quad -(\mathrm{A}_2 e - \mathrm{B}_2)\sin 4\eta\\ &\quad -(\mathrm{A}_3 e - \mathrm{B}_3)\sin 6\eta\\ &\quad -\&c. \qquad\qquad (10)\end{aligned}$$

where

$$\mathrm{A}_n = n\mathrm{R}\,\frac{4q^n}{1+q^{2n}}, \qquad \mathrm{B}_n = \frac{1}{n}\,\frac{4q^n}{1+q^{2n}}.$$

For values of the modular angle that are not very considerable q is very small, and therefore B_1 is small, and as we know that the series

$$A_1 \sin 2\eta + A_2 \sin 4\eta + A_3 \sin 6\eta + \&c.$$

is the sine of the series

$$2\eta + B_1 \sin 2\eta + B_2 \sin 4\eta + \&c.,$$

it is clear that, for values of the modular angle that are not near 90°, A_1 will not differ much from unity; but the following numbers show at a glance the variation of A_1 and q as the modular angle increases.

k	A_1	q
sin 15°	0·99991	0·00433
sin 30°	0·99839	0·01797
sin 45°	0·99071	0·04321
sin 60°	0·96399	0·08580
sin 75°	0·87701	0·16303

Thus, for values of the modular angle less than 45°, the value of A_1 does not differ from unity by the one-hundredth part, and the value of q is less than $\frac{1}{23}$. Speaking roughly, the values of A_n and B_n are respectively equal to $q\,A_{n-1}$ and $q\,B_{n-1}$, so that for all values less than 60° the series (9) is very convergent, while A_1 differs very little from unity. The method would evidently not be a suitable one if the angle were as large as 75°, and in what follows the angle is supposed not to exceed 50° or 60°.

§ 4. The coefficient of sin 2η in (10) can be made to vanish if the modulus k be so chosen that $A_1 e = B_1$; that is, taking $A_1 = 1$, if approximately

$$e = \frac{4q}{1+q^2} = 4q \text{ nearly};$$

or

$$q = \frac{1}{4} e.$$

In order to obtain exactly the order of the first term when q has this value, it is necessary to use the expression for $\left(\frac{k\mathrm{K}}{\pi}\right)^2$ in terms of q, viz.

$$\left(\frac{k\mathrm{K}}{\pi}\right)^2 = \frac{4q}{1-q^2} + \frac{12q^3}{1-q^6} + \frac{20q^5}{1-q^{10}} + \&c.$$

$$= 4q\,(1 + 4q^2 + 6q^4 + \&c.);$$

whence, rejecting powers of q above cubes,

$$R = \frac{1}{4q(1+4q^2)} = \frac{1-4q^2}{4q};$$

so that, when $e = 4q$,

$$Re = 1-4q^2$$

and the coefficient of sin 2η

$$= 4q\left(\frac{1-4q^2}{1+q^2} - \frac{1}{1+q^2}\right)$$

$$= -16q^3 = -\frac{1}{4}e^3;$$

and thus, retaining only the terms of the highest order in e in each coefficient, the equation (10) becomes

$$M = 2\eta + \frac{1}{4}e^3 \sin 2\eta - \left(2-\frac{1}{2}\right)\frac{1}{4}e^2 \sin 4\eta - \left(3-\frac{1}{3}\right)\frac{1}{4^2}e^3 \sin 6\eta$$

$$-\left(4-\frac{1}{4}\right)\frac{1}{4^3}e^4 \sin 8\eta - \&c.$$

$$= 2\eta + \frac{1}{4}e^3 \sin 2\eta - \frac{3}{8}e^2 \sin 4\eta - \frac{1}{6}e^3 \sin 6\eta - \frac{15}{256}e^4 \sin 8\eta - \&c.;$$

so that M differs from 2η only by quantities of the order e^2.

The value of q for which the coefficient of sin 2η exactly vanishes is, as is evident at sight from (9), given by the equation

$$e = \frac{1}{R} = \left(\frac{k\mathrm{K}}{\pi}\right)^2,$$

that is, by

$$e = 4q + 16q^3 + 96q^5 + \&c.,$$

giving

$$q = \frac{1}{4}e - \frac{1}{16}e^3 + \frac{21}{512}e^5 - \&c.$$

and therefore $q = \frac{1}{4}e$ is a very close approximation.

§ 5. The special point of advantage that the elliptic function solution offers is now apparent, viz. in the equation $M = E - e \sin E$ we replace E and sin E by two series, the former being of the form $2\eta + B_1 \sin 2\eta + B_2 \sin 4\eta + \&c.$, and the latter of the form $A_1 \sin 2\eta + A_2 \sin 4\eta + \&c.$, but we have at our disposal a certain quantity (the modulus) on which the values of A_1, $A_2 \ldots$ B_1, $B_2 \ldots$ depend, and we can so select it that the coefficient of sin 2η in the combined expression for M may be very small, and also the coefficients of the succeeding terms.

In his solution Dr. Gyldén has not chosen his modulus with this view, but, as mentioned in § 1, he takes

$$k = \sqrt{\left(\frac{2e}{1+e}\right)},$$

this value being assigned to k, not in order that the coefficients in (10) may be rendered as small as possible, but that the equation (1), viz.—

$$\tan\frac{1}{2}f = \sqrt{\left(\frac{1+e}{1-e}\right)}\tan\frac{1}{2}\mathrm{E}$$

may, by comparison with the elliptic function relation

$$k'\tan\mathrm{am}\,u = \frac{1}{\tan\mathrm{coam}\,u},$$

be converted into the relation

$$\mathrm{coam}\,u = 90^\circ - \mathrm{am}\,v.$$

Thus the object in view in determining the modulus was to obtain an analytical transformation of the problem of two bodies; but if the solution of Kepler's problem be alone considered—and to this Dr. Gyldén restricts himself in the numerical examples that he gives in his paper—there seems no reason why the modulus should not be selected so as to render as rapid as possible the approximations. If we calculate E directly from the equation $\mathrm{M} = \mathrm{E} - e\sin\mathrm{E}$ we have the case corresponding to $k = 0$: here there is only one term to be calculated at each approximation, but it is large, viz. of the order e. When, instead, we replace E and sin E by series, we have more terms to calculate at each approximation, but then we can so determine k that the largest term may be of the order e^2. If the value of k so obtained (that is from the equation $4q = e$) gave so large a value of q that very many terms had to be included, it would be necessary to consider whether it would not be desirable to give some smaller value to q, even though the magnitude of the largest coefficient would thus be increased. It will be found, however, that for all the values of e for which the method is suitable, the value of q is sufficiently small to render the convergence of the terms rapid, so that the assumption $4q = e$ need not be modified on this account.

§ 6. As the equation $4q = e$ is only obtained as an approximation to the equation $\mathrm{A}_1 e = \mathrm{B}_1$, it is clear that there is no advantage in its being *exactly* satisfied, and it will generally be amply sufficient if the modular angle be taken equal to the exact number of degrees nearest to the angle for which $4q = e$.

The following Table gives the values of the coefficients

$$\mathrm{A}_1 = \frac{\pi^2}{k^2\mathrm{K}^2}\frac{4q}{1+q^2}, \quad \mathrm{B}_1 = \frac{4q}{1+q^2},$$

and also the values of q and $\log q$ for every degree of the modular angle. The values for angles at all near 90° would not, of course, be used, but it seems desirable to give the Table in its integrity.

Table I.

Arc sin k	A_1	B_1	q	$\log q$
°				
1	1·0000000	0·0000762	0·00001904	5·27965680
2	1·0000000	0·0003047	0·00007617	5·88178296
3	0·9999998	0·0006857	0·00017143	6·23407579
4	0·9999995	0·0012195	0·00030487	6·48410778
5	0·9999989	0·0019063	0·00047657	6·67812662
6	0·9999976	0·0027466	0·00068665	6·83673234
7	0·9999954	0·0037409	0·00093522	6·97091369
8	0·9999924	0·0048898	0·00122245	7·08723006
9	0·9999882	0·0061940	0·00154850	7·18991248
10	0·9999817	0·0076543	0·00191359	7·28184993
11	0·9999730	0·0092717	0·00231795	7·36510313
12	0·9999619	0·0110472	0·00276181	7·44119369
13	0·9999476	0·0129817	0·00324547	7·51127726
14	0·9999289	0·0150767	0·00376923	7·57625221
15	0·9999059	0·0173334	0·00433342	7·63683083
16	0·9998783	0·0197532	0·00493841	7·69358743
17	0·9998441	0·0223377	0·00558460	7·74699184
18	0·9998034	0·0250886	0·00627239	7·79743338
19	0·9997550	0·0280077	0·00700226	7·84523825
20	0·9996982	0·0310968	0·00777468	7·89068255
21	0·9996311	0·0343582	0·00859018	7·93400202
22	0·9995537	0·0377938	0·00944930	7·97539964
23	0·9994644	0·0414061	0·01035265	8·01505137
24	0·9993617	0·0451976	0·01130084	8·05311085
25	0·9992446	0·0491708	0·01229456	8·08971301
26	0·9991115	0·0533285	0·01333451	8·12497701
27	0·9989607	0·0576738	0·01442144	8·15900867
28	0·9987908	0·0622096	0·01555616	8·19190236
29	0·9986000	0·0669393	0·01673951	8·22374268
30	0·9983865	0·0718663	0·01797239	8·25460576
31	0·9981482	0·0769944	0·01925575	8·28456038
32	0·9978827	0·0823275	0·02059060	8·31366893
33	0·9975884	0·0878696	0·02197800	8·34198819
34	0·9972621	0·0936250	0·02341909	8·36957004
35	0·9969019	0·0995984	0·02491506	8·39646198

Arc sin k	A_1	B_1	q	log q
36°	0·9965046	0·1057946	0·02646718	8·42270772
37	0·9960677	0·1122187	0·02807680	8·44834754
38	0·9955875	0·1188761	0·02974532	8·47341870
39	0·9950616	0·1257725	0·03147428	8·49795576
40	0·9944852	0·1329139	0·03326526	8·52199088
41	0·9938557	0·1403068	0·03511996	8·54555404
42	0·9931682	0·1479578	0·03704020	8.56867332
43	0·9924188	0·1558741	0·03902789	8·59137506
44	0·9916026	0·1640633	0·04108507	8·61368404
45	0·9907150	0·1725335	0·04321392	8·63562365
46	0·9897501	0·1812931	0·04541675	8·65721604
47	0·9887027	0·1903511	0·04769603	8·67848227
48	0·9875664	0·1997172	0·05005441	8·69944237
49	0·9863346	0·2094017	0·05249471	8·72011550
50	0·9850005	0·2194155	0·05501993	8·74052006
51	0·9835562	0·2297702	0·05763334	8·76067376
52	0·9819934	0·2404780	0·06033839	8·78059371
53	0·9803034	0·2515525	0·06313883	8·80029653
54	0.9784766	0·2630077	0·06603869	8·81979842
55	0·9765026	0·2748590	0·06904230	8·83911525
56	0·9743697	0·2871227	0·07215437	8·85826262
57	0·9720666	0·2998163	0·07537997	8·87725598
58	0·9695790	0·3129590	0·07872464	8·89611069
59	0·9668930	0·3265712	0·08219438	8·91484211
60	0·9639930	0·3406752	0·08579573	8·93346569
61	0·9608609	0·3552952	0·08953588	8·95199709
62	0·9574782	0·3704574	0·09342267	8·97045226
63	0·9538241	0·3861905	0·09746475	8·98884758
64	0·9498751	0·4025257	0·10167168	9·00719999
65	0·9456061	0·4194977	0·10605402	9·02552714
66	0·9409892	0·4371446	0·11062354	9·04384755
67	0·9359923	0·4555081	0·11539337	9·06218085
68	0·9305806	0·4746350	0·12037825	9·08054801
69	0·9247144	0·4945775	0·12559479	9·09897161
70	0·9183501	0·5153942	0·13106182	9·11747621
71	0·9114365	0·5371509	0·13680085	9·13608879
72	0·9039167	0·5599223	0·14283652	9·15483926
73	0·8957248	0·5837942	0·14919737	9·17376116
74	0·8867853	0·6088652	0·15591666	9·19289252
75	0·8770097	0·6352492	0·16303353	9·21227694

Arc sin k	A_1	B_1	q	log q
76°	0·8662938	0·6630809	0·17059454	9·23196512
77	0·8545150	0·6925190	0·17865566	9·25201679
78	0·8415247	0·7237544	0·18728518	9·27250342
79	0·8271432	0·7570202	0·19656766	9·29351207
80	0·8111467	0·7926045	0·20660976	9·31515082
81	0·7932526	0·8308727	0·21754895	9·33755699
82	0·7730953	0·8722976	0·22956716	9·36090976
83	0·7501866	0·9175126	0·24291297	9·38545071
84	0·7238497	0·9673968	0·25794020	9·41151903
85	0·6930978	1·0232358	0·27517980	9·43961656
86	0·6563906	1·087040	0·29548839	9·47054042
87	0·6110736	1·162285	0·32040034	9·50569296
88	0·5517805	1·256006	0·35316565	9·54797845
89	0·4637601	1·387542	0·40330931	9·60563824

By means of this Table we can always, for any given value of e (supposed less than $\frac{1}{2}$) at once select that value of the modular angle for which the value of $A_1 e - B_1$ is the least. In the formation of this Table, eight-decimal values of log q and log K were used, and log A_1 and log B_1 are given to eight places (except above 85°) in Table II., at the end of the paper. The values of A_1 and B_1 were deduced from those of log A_1 and log B_1 in Table II. by means of ordinary seven-figure logarithms, so that the last figures are not to be relied upon; this applies of course especially to the numbers in the A_1 column, which, commencing generally with nines, were taken out from near the end of the table of logarithms. The values of q, however, were obtained by ten-figure logarithms, and should be correct to the last figure.

§ 7. I take as an example the case of $e = \frac{1}{4}$, M = 50°, which is the first of the two examples which Dr. Gyldén has given in his paper. We see at once from the foregoing Table that B_1 is most nearly equal to $\frac{1}{4}A_1$ when $k = \sin 52°$; and taking this value of the modulus and working with seven-figure logarithms, I find

$A_1 = 0{\cdot}9819934$ $\quad$ $B_1 = 0{\cdot}2404780$

$A_2 = 0{\cdot}1189336$ $\quad$ $B_2 = 0{\cdot}0072813$

$A_3 = 0{\cdot}0107645$ $\quad$ $B_3 = 0{\cdot}0002929$

$A_4 = 0{\cdot}0008660$ $\quad$ $B_4 = 0{\cdot}0000133$

$A_5 = 0{\cdot}0000653$ $\quad$ $B_5 = 0{\cdot}0000006$

$A_6 = 0{\cdot}0000047$

$A_7 = 0{\cdot}0000003$

whence, e being $= \frac{1}{4}$,

$$\begin{aligned} 2\eta = \mathrm{M} &+ 0{\cdot}0050203 \sin 2\eta \\ &+ 0{\cdot}0224521 \sin 4\eta \\ &+ 0{\cdot}0023982 \sin 6\eta \\ &+ 0{\cdot}0002032 \sin 8\eta \\ &+ 0{\cdot}0000157 \sin 10\eta \\ &+ 0{\cdot}0000012 \sin 12\eta \\ &+ 0{\cdot}0000001 \sin 14\eta \\ &+ \&c. \end{aligned}$$

and the formulæ are

$$\begin{aligned} 2\eta = \mathrm{M} &+ [3{\cdot}01515] \sin 2\eta \\ &+ [3{\cdot}665682] \sin 4\eta \\ &+ [2{\cdot}6943] \sin 6\eta \\ &+ [1{\cdot}6223] \sin 8\eta \\ &+ [0{\cdot}510] \sin 10\eta \\ &+ [9{\cdot}394] \sin 12\eta \\ &+ [8{\cdot}31] \sin 14\eta \\ &+ \&c. \end{aligned} \qquad \begin{aligned} \mathrm{E} = 2\eta &+ [4{\cdot}6955006] \sin 2\eta \\ &+ [3{\cdot}176637] \sin 4\eta \\ &+ [1{\cdot}7811] \sin 6\eta \\ &+ [0{\cdot}437] \sin 8\eta \\ &+ [9{\cdot}120] \sin 10\eta \\ &+ \&c. \end{aligned}$$

. . . (11)

where the numbers in [ ] denote the Briggian logarithms of the coefficients expressed in seconds of arc.

When M is put $= 50°$ the successive approximations to 2η are found to be

°	′	″
50		
51	33	4
51	31	51·4
51	31	52·4
51	31	52·4

whence, from the formula for 2 am v, $=$ E, it results that

$$\mathrm{E} = 62° \ 43' \ 55''{\cdot}9.$$

§8. In Dr. Gyldén's paper the process is as follows :—

$$e = \frac{1}{4}, \text{ whence } k^2 = \frac{2}{5}, \quad k'^2 = \frac{3}{5},$$

and using the formula $q = \lambda + 2\lambda^5 + \&c.$ where

$$\lambda = \frac{1}{2} \frac{k^2}{(1+k')(1+\sqrt{k'})^2},$$

it is shown that

$$\log q = 8{\cdot}503565 ;$$

whence, from the formula

$$\frac{2K}{\pi} = (1 + 2q + 2q^4 + 2q^9 + \&c.)^2$$

$$\log \frac{2K}{\pi} = 0{\cdot}053694.$$

It is thus found that

$$\begin{aligned} 2\eta = M &+ [4{\cdot}398392] \sin 2\eta \\ &+ [3{\cdot}455685] \sin 4\eta \\ &+ [2{\cdot}16940] \sin 6\eta \\ &+ [0{\cdot}799] \sin 8\eta \\ &+ [9{\cdot}415] \sin 10\eta \\ &+ \&c. \end{aligned} \qquad \begin{aligned} 2\,\mathrm{am}\,v = 2\eta &+ [4{\cdot}419607] \sin 2\eta \\ &+ [2{\cdot}62258] \sin 4\eta \\ &+ [0{\cdot}950] \sin 6\eta \\ &+ [9{\cdot}319] \sin 8\eta \\ &+ \&c. \end{aligned} \quad \ldots \quad (12)$$

Dr. Gyldén starts with $2\eta = 56^\circ$ as a first approximation, and finds the successive approximations to be

°	′	″
56	32	4
56	32	6·2

giving

$$E, = 2\,\mathrm{am}\,v, = 62^\circ\ 43'\ 56''{\cdot}0.$$

But in order to compare the approximations by means of (12) with that obtained from (11), it is necessary to commence the approximation with $2\eta = 50^\circ$. The approximations thus obtained are shown below in the second column; and for the sake of comparison those derived by the direct determination of E from the equation

$$E = M + \tfrac{1}{4} \sin E,$$

that is from

$$E = M + [4{\cdot}7123651] \sin E \quad \ldots \quad (13)$$

are added in the third column.

$$e = \tfrac{1}{4}, \qquad M = 50^\circ.$$

Approximations obtained from (11).			Approximations obtained from (12).			Approximations obtained from (13).		
°	′	″	°	′	″	°	′	″
50			50			50		
51	33	4	56	7	35	60	58	22
51	31	51·4	56	30	46	62	31	29
51	31	52·4	56	32	2	62	42	30
51	31	52·4	56	32	6·1	62	43	46
giving			56	32	6·3	62	43	54·8
$E = 62$	43	55·9	56	32	6·3	62	43	55·8
			giving			62	43	55·9
			$E = 62$	43	56·0	62	43	55·9

Thus, not counting the first approximation, viz. 50°, we obtain the final value from (11) in three approximations, from (12) in five, and from (13) in seven. The difference of a tenth of a second in the values of E is unimportant, as a last-figure error of a unit is of course quite possible. It was not necessary to calculate the earlier approximations to seconds, but it was convenient for the sake of uniformity to carry all the approximations to this extent.

§ 9. Dr. Gyldén's value of k is $\sqrt{\left(\frac{2}{5}\right)}$ corresponding to a modular angle of 39° 14′, while the modular angle in (11) was 52°. The corresponding values of q are 0·0318834 and 0·0603384: but only one more term is required in (11) than in (12).

On calculating the coefficient of sin 2η with Dr. Gyldén's value of q, I find it to be

$$0{\cdot}994935\, e - 0{\cdot}127404$$

which, putting $e = \frac{1}{4}$, becomes

$$0{\cdot}248734 - 0{\cdot}127404$$

so that in this case the subtraction of the second series only diminishes the coefficient of sin 2η in the first by about one-half: and the whole coefficient is equal to about one-half of e.

Expressed in seconds the values of the coefficients of the terms in (11) and (12) are

	In (11).	In (12).
	"	"
Sin 2η	1035·5	25026·0
Sin 4η	4631·1	2855·5
Sin 6η	494·7	147·7
Sin 8η	41·9	6·3
Sin 10η	3·2	0·3
Sin 12η	0·2	

and the sum of all the coefficients in (11) is 6207″ or 1° 43′ 27″, and in (12) 28036″ or 7° 47′ 16″. The coefficient in (13) is 51566″ or 14° 19′ 26″.

Another advantage that results from obtaining the modular angle from the Table in § 6, instead of determining it as an analytical function of e, is that in the former case all the necessary tables can be calculated beforehand. It was this, in fact, which, when reading Dr. Gyldén's paper, first led me to examine the subject, as the existence of an exact analytical relation between q and e required that the computer should himself calculate, for each given value of e, the values of K and q; and this would be avoided if use could be made, instead, of a small table of these quantities. In the practical application of the method, it certainly would be an important gain, both in

point of labour and accuracy, to have the K's and q's given, as the necessity for the calculation of them by the user of the method seemed of itself to form a serious objection to its employment, except in those cases where so many values of E were required that the preliminary calculations might be relatively disregarded.

§ 10. By means of the equation $q = \frac{1}{4}e$ we can of course express k in terms of e, though the general term cannot be exhibited, as there is no q-series for k. Starting, however, with the series for $\log k$, viz.:

$$\log k = \log 4\sqrt{q} - \frac{4q}{1+q} + \frac{4q^2}{2(1+q^2)} - \frac{4q^3}{3(1+q^3)} + \&c.$$

$$= \log 4\sqrt{q} - 4q + 6q^2 - \frac{16}{3}q^3 + 3q^4 - \&c.,$$

we have

$$k = 4\sqrt{q}.e^{-4q+6q^2-\&c.}$$

$$= 4\sqrt{q}\{1 - 4q + 14q^2 - 40q^3 + 101q^4 - \&c.\}$$

$$= 2\sqrt{e}\{1 - e + \frac{7}{8}e^2 - \frac{5}{8}e^3 + \frac{101}{256}e^4 - \&c.\},$$

Dr. Gyldén's value being

$$k = \sqrt{\left(\frac{2e}{1+e}\right)}$$

$$= \sqrt{(2e)}\{1 - \frac{1}{2}e + \frac{3}{8}e^2 - \frac{5}{16}e^3 + \&c.\}.$$

Thus the former series gives the greater value of q; but the following table shows better the amount of difference between the formulæ:

	(α) $q = \frac{1}{4}e$	(β) $k = \sqrt{\left(\frac{2e}{1+e}\right)}$			
e	k from (α)	k from (β)	e	k from (α)	k from (β)
	°	° ′		° ′	° ′
$\frac{1}{20}$	25	17 59	$\frac{5}{20}$	52	39 14
$\frac{2}{20}$	35	25 14	$\frac{6}{20}$	56	42 48
$\frac{3}{20}$	42	30 43	$\frac{7}{20}$	60	46 4
$\frac{4}{20}$	48	35 16	$\frac{8}{20}$	63	49 6

§ 11. Dr. Gyldén seems, as stated in § 5, to have been led to the value of the modulus which he adopted by analytical considerations, which have reference to the treatment of the general question of elliptic motion; while in this paper the object has

been to reduce, as far as possible, the numerical calculations required for the solution of Kepler's problem. I have confined myself entirely to the solution of Kepler's problem—that is, to the determination of E from the equation $E = M + e \sin E$ by means of elliptic functions—and though it is true that Dr. Gyldén does so too in the examples in his paper, he does not appear to have specially considered the solution of this question apart from the more general investigation. If the true anomaly f be required in terms of M, probably the best method would in every case be to obtain E from M and then to deduce f from E by the trigonometrical formula (1).

The q-formula for tan am$v$ is

$$\frac{\pi}{2k'K}\{\tan\eta - \frac{4q^2}{1+q^2}\sin 2\eta + \frac{4q^4}{1+q^4}\sin 4\eta - \&c.\},$$

and it is clear that since the q-coefficients are different from those in the series for am$v$ or sn$v$ cn$v$, it would be most convenient to first calculate E, $= 2\,\mathrm{am}v$, and then have recourse to (1) to obtain f. Also the form into which (1) is transformed, viz. $\mathrm{coam}u = 90° - \mathrm{am}v$, that is, $\mathrm{am}(K-u) = 90° - \mathrm{am}v$, or, as we may write it,

$$K - u = F(90° - \tfrac{1}{2}E, k),$$

is not preferable to (1) for the calculation of u or f; so that, as far as the numerical solution of Kepler's problem is concerned, the value of k given by (3) does not present any special advantages.

§ 12. The other example which Dr. Gyldén takes is $e = \frac{1}{2}$, $M = 60°$. It is not a very satisfactory one for the method in any case, as e is rather too large; and it is even less favourable as an example of the process of solution suggested in this paper, for in the reasoning in §§ 4 and 5 e^2 has been supposed to be considerably less than e, and this is not the case if e be as large as $\frac{1}{2}$.

In this case the Table in § 6 shows that $A_1 e - B_1$ is smallest when $k = \sin 68°$, and the formulæ are

$$\begin{aligned}
2\eta = M &- [3{\cdot}28499]\sin 2\eta \\
&+ [4{\cdot}242028]\sin 4\eta \\
&+ [3{\cdot}57437]\sin 6\eta \\
&+ [2{\cdot}8035]\sin 8\eta \\
&+ [1{\cdot}9915]\sin 10\eta \\
&+ [1{\cdot}1568]\sin 12\eta \\
&+ [0{\cdot}308]\sin 14\eta \\
&+ [9{\cdot}455]\sin 16\eta \\
&+ [8{\cdot}59]\sin 18\eta \\
&+ \&c.
\end{aligned}$$

$$\begin{aligned}
2\,\mathrm{am}v = 2\eta &+ [4{\cdot}9907850]\sin 2\eta \\
&+ [3{\cdot}77646]\sin 4\eta \\
&+ [2{\cdot}68101]\sin 6\eta \\
&+ [1{\cdot}6366]\sin 8\eta \\
&+ [0{\cdot}620]\sin 10\eta \\
&+ [9{\cdot}622]\sin 12\eta \\
&+ [8{\cdot}64]\sin 14\eta \\
&+ \&c.
\end{aligned}$$

. . . (14)

Dr. Gyldén has

$$k = \sqrt{\binom{2}{3}} = \sin 54^\circ\, 44',$$

and his formulæ are

$$\begin{aligned} 2\eta = \mathrm{M} &+ [4{\cdot}650549] \sin 2\eta \\ &+ [4{\cdot}075356] \sin 4\eta \\ &+ [3{\cdot}12280] \sin 6\eta \\ &+ [2{\cdot}0940] \sin 8\eta \\ &+ [1{\cdot}029] \sin 10\eta \\ &+ [9{\cdot}945] \sin 12\eta \\ &+ [8{\cdot}86] \sin 14\eta \\ &+ \&c. \end{aligned} \qquad \begin{aligned} 2\,\mathrm{am}\,v = 2\eta &+ [4{\cdot}748494] \sin 2\eta \\ &+ [3{\cdot}283498] \sin 4\eta \\ &+ [1{\cdot}9415] \sin 6\eta \\ &+ [0{\cdot}651] \sin 8\eta \\ &+ [9{\cdot}387] \sin 10\eta \\ &+ \&c. \end{aligned} \quad \ldots \quad (15)$$

while the original equation $\mathrm{E} = \mathrm{M} + e \sin \mathrm{E}$ becomes

$$\mathrm{E} = \mathrm{M} + [5{\cdot}0133951] \sin \mathrm{E} \quad \ldots \quad (16)$$

The approximations derived from these three formulæ are shown below:—

$$e = \tfrac{1}{2}, \quad \mathrm{M} = 60^\circ.$$

From (14).	From (15).	From (16).
° ′ ″	° ′ ″	° ′ ″
60	60	60
63 34	73 36	84 49
63 0 26	73 26 15	88 32
63 5 48·6	73 26 39·4	88 38 19
63 4 57·2	73 26 38·3	88 38 23·3
63 5 5·5	73 26 38·4	88 38 23·3
63 5 4·1	73 26 38·4*	
63 5 4·3	giving	
63 5 4·3	E = 88 38 23·2	
giving		
E = 88 38 23·4		

Thus, using (14) there are seven approximations, using (15) five, and using (16) only four. The reason of this is that the greatest term in (15) is the first, viz. that which involves $\sin 2\eta$, and as 2η in this example is equal to about 73°, a small change in the angle produces but a slight alteration in the sine, and the approximation is rapid. It is even more so in (16), where the

* I have recalculated and slightly altered Dr. Gyldén's last approximations, so that all three sets should be exactly similar to one another. Dr. Gyldén's last two approximations are 73° 26′ 39″·4 and 73° 26′ 38″·2, giving E = 88° 38′ 23″·0.

angle is very nearly equal to 90°. But in (14) the greatest term is that which involves sin 4η, and 4η is equal to about 126°, which is not far distant from the middle of the quadrant, so that a change in 2η produces a considerable alteration in the value of sin 4η. All this will be reversed when 4η in (14) is nearer to 90° than 2η in (16). Take for example $M = 30°$ and the approximations are as follows:—

$$e = \frac{1}{2}, \quad M = 30°.$$

From (14).	From (15).	From (16).
° ′ ″	° ′ ″	° ′ ″
30	30	30
35 8	39 28	44 19
35 22 25	41 28 32	50 1
35 22 44·0	41 49 5	51 57 3
35 22 44·3	41 52 24·6	52 33 35
35 22 44·3	41 52 56·6	52 44 46
giving	41 53 1·8	52 48 9
E = 52 49 37·6	41 53 2·6	52 49 11
	41 53 2·8	52 49 29
	41 53 2·8	52 49 34·9
	giving	52 49 36·7
	E = 52 49 37·6	52 49 37·2
		52 49 37·4
		52 49 37·5
		52 49 37·5

Here 4η in (14) is equal to about 71°, and 2η in (15) to about 42°; and as E is only about 53° the approximation by (16) is not very rapid—in fact, by (14) there are four approximations, by (15) eight, and by (16) thirteen.

In this case of $e = \frac{1}{2}$ there is not much to choose between the two formulæ, as the reasoning in § 4 is not appropriate when e is so large: for the object was to cause the term of the first order in e to nearly disappear, the next term (in which the coefficient was approximately $= \frac{3}{8}e^2$) being the largest; but as this term involves sin 4η, a small change in the value of 2η produces double the effect that it would if sin 2η were involved instead; so that when $e = \frac{1}{2}$ the advantage arising from the term being of the second order is nearly neutralised. This is pretty evident; but it may be seen perhaps more clearly by expanding x by Lagrange's theorem (i) when $x = M + e \sin x$ and (ii) when $x = M + e \sin 2x$.

The values of the coefficients of the terms in (14) and (15) are

	In (14).	In (15).
	"	"
sin 2η	1927·5	44724·9
sin 4η	17459·3	11894·8
sin 6η	3752·9	1326·8
sin 8η	636·1	124·2
sin 10η	98·1	10·7
sin 12η	14·3	0·9
sin 14η	2·0	0·1
sin 16η	0·3	

and the sum of all the coefficients in (14) is 23890″ or 6° 38′ 10″, and in (15) is 58082″ or 16° 8′ 2″. Only one more term is required in (14) than in (15), as the term in sin 18η can never be sensible. The coefficient in (16) is equal to 103132″ or 28° 38′ 52″.

§ 13. In regard to the practical use of the method Dr. Gyldén remarks, and very truly, that the calculations are much less laborious than at first sight they appear to be, as only approximate values of the arguments are required in the smaller terms. Also when 2η is found the value of 2 am v is obtained with great ease, as the values of log sin 2η, log sin 4η, &c., have been already calculated. The obvious objection to the method is the great number of logarithmic entries that are required: and this objection seems so serious as almost to be fatal to its adoption. As far as the three examples given in this paper are concerned, the old trigonometrical method is certainly preferable; but this might not be the case perhaps if the value of E were smaller, say not exceeding 30° or 40°, when the number of approximations would be considerable. I must admit that while working out the examples in §§7, 8, and 12, the conclusion rather forced itself upon me that the method was too complicated and elaborate for ordinary practical use, and that some of the numerous other methods that have been given for the solution of Kepler's problem must be superior to it. I have, however, made no other numerical comparisons besides those described in this paper; and Dr. Gyldén's words seem to imply that he has found the mode of solution proposed by him to be convenient and useful. If a more complete examination should render it probable that the elliptic function method might be of practical value, I would calculate the necessary tables of the values of the second and succeeding terms. My intention was to have appended them to this paper, and some of the calculations were performed; but, as the above comparisons tended rather to show that the method was not a very good one in practice, they have been laid aside: it seems worth while, however, to give the values of $\log A_1$ and $\log B_1$ for every degree, which are contained in Table II., and the values of the quantities involved in

the coefficients of the second and third terms, viz., A_2, B_2, A_3, B_3, for every fifth degree, which are contained in Table III.; as these may be found useful in any further discussion of the question. Table III. was obtained by means of seven-figure logarithms, so that the last figures are liable to small errors. Table II. is referred to at the end of §6. I am in great hopes that Dr. Gyldén or some other astronomer who has used the method may be tempted by this paper to make some further remarks in regard to it.

Whether the elliptic function method should be found to be practically important or not, the substitution for E and sin E of two series proceeding by multiples of 2η and whose coefficients depend upon an arbitrary quantity k is a remarkable transformation of the question, and the form assumed by the equation $M = E - e \sin E$, viz.

$$M = 2\left(\mathrm{am}\, v + \frac{e}{k^2}\frac{d^2}{dv^2}\mathrm{am}\, v\right),$$

is noticeable, for, as a consequence of it, the q-coefficients of the two series are identical, to a factor *près*. It certainly seemed desirable to examine the effect of these transformations: and the practical advantages and defects of the method could not well have been exhibited otherwise than by means of numerical examples.

I ought to mention that Dr. Gyldén only gives the method discussed in this paper as appropriate when e does not exceed $\frac{1}{2}$: he proposes a different mode of solution by means of elliptic functions for the case of $e > \frac{1}{2}$, which I have not examined.

Table II.

Arc sin k	log A_1	log B_1
°		
1	9·99999999	5·88171679
2	9·99999999	6·48384295
3	9·99999996	6·83613578
4	9·99999983	7·08616777
5	9·99999950	7·28018652
6	9·99999895	7·43879211
	9·99999809	7·57297329
8	9·99999676	7·68928940
9	9·99999479	7·79197143
10	9·99999202	7·88390832
11	9·99998831	7·96716078
12	9·99998344	8·04325038

Arc sin k	log A_1	log B_1
°		
13	9·99997715	8·11333269
14	9·99996913	8·17830603
15	9·99995919	8·23888265
16	9·99994702	8·29563682
17	9·99993226	8·34903828
18	9·99991458	8·39947630
19	9·99989354	8·44727696
20	9·99986877	8·49271631
21	9·99983976	8·53602996
22	9·99980611	8·57742085
23	9·99976726	8·61706480
24	9·99972271	8·65511539
25	9·99967176	8·69170734
26	9·99961392	8·72695978
27	9·99954842	8·76097834
28	9·99947456	8·79385726
29	9·99939161	8·82568099
30	9·99929872	8·85652550
31	9·99919498	8·88645936
32	9·99907951	8·91554481
33	9·99895137	8·94383847
34	9·99880934	8·97139188
35	9·99865242	8·99825244
36	9·99847937	9·02446359
37	9·99828889	9·05006531
38	9·99807953	9·07509459
39	9·99784995	9·09958575
40	9·99759841	9·12357055
41	9·99732333	9·14707871
42	9·99702280	9·17013786
43	9·99669495	9·19277403
44	9·99633768	9·21501156
45	9·99594870	9·23687340
46	9·99552553	9·25838113
47	9·99506569	9·27955541
48	9·99456628	9·30041564
49	9·99402427	9·32098035
50	9·99343640	9·34126734
51	9·99279913	9·36129359

Arc sin k	log A_1	log B_1
52°	9·99210856	9·38107544
53	9·99136048	9·40062864
54	9·99055043	9·41996853
55	9·98967336	9·43910996
56	9·98872383	9·45806741
57	9·98769602	9·47685526
58	9·98658326	9·49548740
59	9·98537851	9·51397791
60	9·98407385	9·53234059
61	9·98266051	9·55058935
62	9·98112889	9·56873825
63	9·97946823	9·58680151
64	9·97766654	9·60479368
65	9·97571033	9·62272967
66	9·97358459	9·64062507
67	9·97127225	9·65849611
68	9·96875396	9·67635984
69	9·96600769	9·69423447
70	9·96300824	9·71213958
71	9·95972642	9·73009630
72	9·95612842	9·74812782
73	9·95217468	9·76625982
74	9·94781856	9·78452110
75	9·94300437	9·80294417
76	9·93766522	9·82156648
77	9·93171963	9·84043165
78	9·92506689	9·85959125
79	9·91758073	9·87910753
80	9·90909936	9·89905660
81	9·89941153	9·91953447
82	9·88823308	9·94066468
83	9·87516937	9·96261199
84	9·85964844	9·98560468
85	9·84079450	0·00997574
86	9·8171624	0·0362456
87	9·7860937	0·0653128
88	9·7417663	0·0989917
89	9·6662934	0·1422460

Table III.

Arc sin k	A_2	B_2	A_3	B_3
°				
5	0·0009531	0·0000005	0·0000007	0·0000000
10	0·0038271	0·0000073	0·0000110	0·0000000
15	0·0086662	0·0000376	0·0000563	0·0000001
20	0·0155456	0·0001209	0·0001813	0·0000006
25	0·0245743	0·0003023	0·0004532	0·0000025
30	0·0358984	0·0006460	0·0009678	0·0000077
35	0·0497066	0·0012415	0·0018577	0·0000206
40	0·0662368	0·0022132	0·0033051	0·0000491
45	0·0857849	0·0037349	0·0055607	0·0001076
50	0·1087164	0·0060543	0·0089724	0·0002221
55	0·1354796	0·0095335	0·0140311	0·0004388
60	0·1666215	0·0147210	0·0214443	0·0008420
65	0·2028009	0·0224921	0·0322658	0·0015904
70	0·2447840	0·0343443	0·0481367	0·0030017
75	0·2933576	0·0531223	0·0717900	0·0057778
80	0·3488540	0·0852199	0·1083035	0·0117586
85	0·4079986	0·1505844	0·1693016	0·0277715

Trinity College, Cambridge,
1877, *March* 20.

Some Experiments Bearing on the Transit of Venus.

By the Rev. S. J. Perry.

A short time since I had the honour of bringing before the Royal Astronomical Society the researches of M. Ch. André on the subject of Instrumental Diffraction in its connection with the Transit of *Venus*, and he has since desired me to supplement my paper by one or two historical remarks:—

1. The main facts adduced by M. André were verifications of results previously obtained by Schwerd, whose theory was completed by André.

2. Professor Kaiser had already pointed out the importance of the question of diffraction in the study of telescopic images.

3. M. Van de Sande Backuysen, successor of Professor Kaiser in the direction of the Leyden Observatory, explained the black drop as a phenomenon of diffraction in a Note addressed to the *Astronomische Nachrichten*, May 11, 1874.

I have lately received from M. Alfred Angot two very interesting papers for presentation to the Society. The subject matter of these communications is one to which he has, in con-

junction with M. André, devoted much time and labour during the last few years. M. Angot was sent as companion to M. André, in the French Government Expedition to Noumea, New Caledonia, to observe the late Transit of *Venus*, and he has since been engaged in measuring the French daguerreotypes and photographs taken on December 8, 1874. I cannot do better than let him communicate his results in his own words.

Etude sur les Images Photographiques obtenues au foyer des Lunettes Astronomiques. *Par* M. Alfred Angot.

On sait depuis longtemps que l'image photographique d'un objet lumineux est dilatée aux dépens des parties obscures ou du champ lui-même, et cela d'autant plus que l'objet est plus vivement éclairé. Pendant longtemps, on s'est borné à attribuer cet effet à un *cheminement* de proche en proche de l'action chimique, sans plus l'étudier. Les applications que l'on tente actuellement de faire de la photographie aux mesures de précision, notamment à celles que demande l'astronomie, rendaient nécessaire l'étude approfondie de ce phénomène.

Le travail qui suit a été effectué dans les caves de l'Ecole Normale Supérieure. Grâce à la bienveillance de M. H. Sainte-Claire Deville, j'ai pu mettre à profit toutes les ressources de son laboratoire, et m'installer dans une cave de 90 mètres de longueur en ligne droite, à côté des appareils qui servaient à la même époque à M. André pour son étude de la diffraction dans les instruments d'optique.

I. *Marche des Expériences.*

Les images photographiques ont été obtenues sans grossissement au foyer d'une lunette que la Commission du Passage de Vénus a bien voulu mettre à ma disposition, et qui m'avait servi à l'observation même du passage à Nouméa (Nouvelle-Calédonie). L'objectif, de $0^{m}{\cdot}13$ d'ouverture, achromatisé par l'écartement des deux lentilles qui le composent, a $3^{m}{\cdot}80$ de distance focale. Une longueur de $1/500^{e}$ de millimètre, mesurée sur les épreuves photographiques, correspond ainsi à un angle de $0''{\cdot}109$.

Les épreuves ont été mesurées avec une des machines construites par MM. Brünner pour la Commission du Passage de Vénus, et qui permettent d'évaluer rapidement et avec certitude le $1/500^{e}$ de millimètre—quantité bien suffisante pour mes recherches.

L'objet photographié, disposé à environ 87 mètres de l'objectif, est une source lumineuse formée de deux rectangles égaux séparés par un intervalle obscur. Ces rectangles sont découpés dans une lame métallique que l'on applique sur une plaque de verre

dépoli, éclairée par derrière avec une lampe Drummond ou la lumière électrique. On réalise ainsi un éclairement uniforme de l'objet photographié—condition absolument indispensable.

Avec cette disposition on peut obtenir facilement, non-seulement le fait de l'augmentation des images, mais la mesure de cette augmentation en valeur absolue. L'augmentation de dimensions de chaque rectangle lumineux est en effet forcément égale à la diminution de l'intervalle obscur compris entre eux : la somme de ces deux quantités est donc constante et égale à ce qu'elle serait si l'image se formait seulement d'après les lois de l'optique géométrique. On connaît de plus, par la mesure directe de l'objet photographié, le rapport des intervalles obscur et lumineux. Possédant leur somme et leur rapport, on calcule facilement leur valeur pour l'image géométrique ; la comparaison avec celles que donne la mesure directe des épreuves donne dans chaque cas, et en valeur absolue, l'augmentation des dimensions de l'image. En outre, la constance de la somme des intervalles obscur et lumineux donne un excellent moyen d'apprécier le degré de précision des expériences.

Pour permettre de juger cette précision, je citerai seulement ici les résultats suivants des mesures de sept images obtenues successivement sur une même plaque daguerrienne, et pour lesquelles on n'a fait varier que la durée d'exposition à la lumière :

Durée de pose	Largeur (en 1/500es de millimètre) du rectangle lumineux l	de l'intervalle obscur o	Somme $l+o$
10 secondes	593·5	192·6	786·1
30 „	618·5	168·6	787·1
40 „	624·0	163·6	787·6
1 minute	632·6	155·2	787·8
2 minutes	645·7	141·4	787·1
4 „	656·4	130·0	786·4
7 „	673·8	113·4	787·2

On voit que l'erreur moyenne sur la somme, qui devrait être constante, ne dépasse pas $0^{mm}\cdot001$ ($0''\cdot05$). Dans ce qui suit, nous nous bornerons à indiquer l'excès de l'image sur l'image géométrique calculée, comme nous l'avons indiqué plus haut, au moyen de la somme $l+o$, et du rapport $\frac{L}{0}$ mesuré directement sur l'objet photographié.

II. *Résultats.*

Les expériences ont montré que la dimension des images varie, 1° avec l'intensité de la lumière ; 2° avec la durée de pose ; 3° avec la sensibilité de la plaque ; 4° avec le diamètre de l'objectif ;

5° avec l'état chimique de la plaque; les phénomènes étant différents si la plaque a vu ou non le jour avant de recevoir l'impression photographique.

1°. *Influence de l'intensité.*—Toutes choses restant égales, l'image augmente à mesure que l'on fait croître l'intensité de la lumière; l'image obtenue est généralement plus grande que l'image géométrique [en supposant un objet lumineux sur fond obscur]. Mais, pour une intensité assez faible, on peut obtenir une image *plus petite* que l'image géométrique. Le phénomène change de sens si l'on considère l'image d'un objet obscur sur champ lumineux.

Je citerai seulement comme exemple l'expérience suivante, qui donne l'excès de l'image obtenue sur l'image géométrique calculée pour des intensités variant de 1 à 38 :

Plaque daguerrienne (durée de pose, 1 minute).

Intensité de la source lumineux	Excès de l'image sur l'image géométrique	
	en 1/500es de millimètre	en secondes d'arc
		"
1	116·1	12·66
1/4	81·0	8·84
1/9	41·5	4·52
1/9·5	39·2	4·27
1/38	−22·4	−2·44

Le signe − devant le dernier nombre indique que l'image obtenue est plus petite que l'image géométrique.

2°. *Influence de la durée de pose.*—La durée de pose a une influence de même ordre que l'intensité; l'image obtenue croît avec elle. J'ai cité plus haut un exemple de cette influence. Le résultat général est qu'il n'y a pas proportionnalité entre la durée de pose et l'intensité, les effets de la durée de pose étant beaucoup moins grands que ceux de cette dernière. Un temps de pose 1 avec une intensité 2 produisent, par exemple, un effet plus grand qu'un temps de pose 2 avec une intensité 1.

3°. *Influence de la sensibilité de la plaque.*—Pour une même durée de pose et une même intensité lumineux les phénomènes s'accroissent avec la sensibilité de la plaque sans qu'il soit cependant possible de donner une loi numérique, la sensibilité n'étant pas définie rigoureusement. Mes expériences permettent, au contraire, de déterminer la sensibilité en la mesurant, par exemple, par l'inverse de l'intensité lumineuse, qui donnera, avec une durée de pose de 1 seconde, une image égale à l'image géométrique.

4°. *Influence du diamètre de l'objectif.*—Pour vérifier la loi de l'ouverture de l'objectif, j'ai réduit celui-ci à moitié, mais en quadruplant alors l'intensité de la lumière, de façon que l'éclat de l'image fût constant, la durée de pose étant alors la même dans les deux cas.

Je citerai seulement les nombres suivants :

		Excès de l'image obtenue sur l'image géométrique en 1/500es de millimètre
Première expérience	ouverture 1	27·0
	„ 1/2	43·0
Deuxième expérience	ouverture 1	50·2
	„ 1/2	72·0

L'image augmente donc quand l'ouverture de l'objectif diminue.

5°. *Influence de l'exposition antérieure à la lumière.*—J'ai exposé à la lumière diffuse une moitié de chaque plaque, et j'ai fait ensuite sur les deux moitiés une série d'épreuves avec la même durée de pose et la même intensité, de façon que toutes les conditions fussent identiques de part et d'autre, sauf l'exposition antérieure à la lumière. L'expérience a montré constamment que l'image obtenue sur la moitié intacte était plus grande que sur celle qui avait préalablement été impressionnée par la lumière diffuse. Je citerai seulement les nombres suivants :

			Largeur de l'image	
		Durée de pose	dans la partie exposée antérieurement à la lumière	dans la partie non exposée
1° plaque daguerrienne		30 secondes	514·9	535·6
		1 minute	537·3	560·4
		4 minutes	563·0	581·5
2° plaque, sur collodion sec	intensité 1	1 minute	584·5	622·7
		2 minutes	620·5	641·0
	intensité environ 1/4	1 minute	516·5	558·0
		2 minutes	558·5	579·0

III. *Théorie.*

La première hypothèse que l'on ait invoquée pour expliquer ces faits était d'admettre un *cheminement* de proche en proche de l'action chimique. Mais il est facile de voir que telle ne saurait être la cause de l'agrandissement observé généralement. Tout d'abord, cette hypothèse ne saurait rendre compte de ce fait, qu'avec une faible intensité ou une courte durée de pose on obtient des images plus petites que l'image géométrique. De plus, s'il y avait un cheminement, il devrait être plus grand sur une plaque ayant déjà subi l'action de la lumière diffuse que sur une autre non altérée, et nous avons vu que l'expérience donnait un résultat précisément contraire. Enfin, en augmentant la durée de pose on devrait accroître le cheminement, en lui donnant plus de temps pour se produire, et une durée de pose 2 avec une intensité 1 donnerait une image plus grande qu'une durée de pose 1 avec

une intensité 2—fait contredit par l'expérience. Il faut donc abandonner cette hypothèse.

La théorie des phénomènes lumineux donne, au contraire, une explication fort simple de tous les phénomènes observés, en admettant seulement, ce qui semble du reste évident, qu'une plaque de sensibilité donnée ne peut être impressionnée pendant le même temps que par des lumières possédant une intensité supérieure à une intensité donnée : cette intensité qui définira justement la sensibilité de la plaque, si l'on peut mesurer en valeur absolue l'intensité limite qui donne une impression pendant l'unité de temps. L'étude de la diffraction au foyer des lentilles faite par Schwerd (*Beugungserscheinungen*), simplifiée et étendue par M. André ('Étude de la Diffraction dans les Instruments d'Optique'—*Annales de l'Ecole Normale Supérieure*, 1876, tome v. page 275), conduit aux résultats suivants. L'intensité lumineuse n'est pas constante en dedans même de l'image géométrique: elle décroît près des bords, et, au bord même, elle n'est que la moitié de ce qu'elle est au centre. En dehors de l'image géométrique on trouve encore de la lumière dont l'intensité décroît progressivement en tendant vers zéro. Si l'on considère un objectif parfait, rigoureusement aplanétique, la largeur de cette zone de diffraction varie en raison inverse de l'ouverture de l'objectif.

L'aberration produit des phénomènes analogues qui se superposent aux précédents et augmentent considérablement l'étendue de la zone diffractée, sans cependant changer le sens général du phénomène. Quand on réduit l'objectif on a toutefois à tenir compte de deux effets inverses ; celui de la diffraction qui augmente, et celui de l'aberration qui diminue en même temps que décroît l'ouverture de l'objectif.

Ces lois suffisent à expliquer tous les faits observés. Quand on augmente l'intensité de la source, l'intensité croît proportionnellement en chacun des points de la zone diffractée, et la plaque photographique, qui reste sensible pour une intensité absolue déterminée, donne des images de plus en plus grandes. Si, au contraire, l'intensité décroît assez pour que, dans le temps considéré, la plaque ne soit plus sensible à la moitié de l'intensité au centre de la plaque, on aura des images plus petites que l'image géométrique.

Au lieu de faire varier l'intensité, on peut faire intervenir la durée de pose ; on aura alors des effets analogues à ceux que donnait la variation de l'intensité, mais moins rapides. En augmentant l'exposition, on rend en effet la plaque sensible à des lumières de plus en plus faibles ; mais cette sensibilité tend rapidement vers une certaine limite, car l'expérience semble prouver que toute intensité qui tombe au-dessous d'une certaine limite, variable avec la sensibilité de la plaque, ne peut plus produire d'impression même pendant un temps infiniment grand.

L'influence de l'ouverture de l'objectif est également manifeste : la zone diffractée augmente quand l'ouverture de l'objectif diminue, et exactement en raison inverse de celle-ci. L'expérience

a montré cette augmentation, mais un peu moins rapide que ne le veut la théorie ; en effet, en même temps que la diminution de l'ouverture produit une augmentation de la zone diffractée, elle diminue notablement l'effet de l'aberration.

Enfin, quand on soumet à la lumière diffuse la plaque sensible avant d'y imprimer l'image, celle-ci vient se faire sur un champ déjà éclairé, et cet éclairement général masque une partie de l'agrandissement diffractionnel.

En résumé, les faits observés sont en parfait accord avec la théorie. La dimension des images photographiques varie avec toutes les conditions de l'expérience ; mais, pour expliquer ces variations, il suffit d'avoir recours aux théories ordinaires de l'optique, et non à l'hypothèse d'un cheminement mystérieux des actions chimiques. Dans une prochaine communication, j'exposerai les applications de ces recherches à l'observation photographique de certains phénomènes astronomiques, notamment à celle du passage de Vénus.

Sur l'Application de la Photographie à l'Observation du Passage de Vénus. *Par* M. Alfred Angot.

Dans un précédent travail, j'ai étudié les causes diverses qui peuvent modifier la dimension des images photographiques, et montré que tous les faits observés sont d'accord avec la théorie de la diffraction. Je désire exposer aujourd'hui les résultats auxquels conduisent mes expériences pour l'application de la photographie à l'observation du passage de *Vénus*.

Avec la méthode d'observation photographique, on peut chercher à déterminer par deux procédés différents la parallaxe solaire au moyen du passage de *Vénus*. 1°. On cherche à mesurer l'effet parallactique direct, c'est-à-dire la différence des positions apparentes de *Vénus* vue au même instant de deux stations éloignées. 2°. On détermine seulement par la photographie l'instant des contacts. Ce dernier procédé conduit identiquement aux mêmes calculs que l'observation directe, qu'il ne fait que doubler. Le premier, au contraire, est propre à la photographie, puisque les astronomes n'ont pas encore imaginé de moyens sûrs de mesurer les distances de *Vénus* et du Soleil à un instant quelconque.

I. *Mesure directe de l'Effet Parallactique.*

La position de *Vénus* sur le Soleil est définie par deux éléments : la distance des centres et l'angle que fait la ligne des centres avec une direction fixe, celle de l'équateur, par exemple, angle que l'on désigne communément sous le nom *d'angle de*

position. Mais, pour déterminer la valeur de la parallaxe, il suffit de connaître les valeurs d'une seule de ces deux quantités mesurées au même moment en deux stations convenablement choisies. De là deux manières de mesurer directement l'effet parallactique :—1°, par l'angle de position ; 2°, par la distance des centres. Nous allons examiner successivement ces deux procédés.

1°. *Mesure de l'angle de position.*—D'après les phénomènes que nous avons décrits dans la première partie de ce travail, il est évident que les diamètres photographiques du Soleil et de *Vénus* seront variables avec toutes les conditions de l'expérience, intensité de la lumière, transparence de l'atmosphère, durée de pose, sensibilité de la plaque, etc. Mais si l'objectif que l'on emploie est bon au point de vue optique, c'est-à-dire ne déforme pas irrégulièrement les images, tous les diamètres du Soleil et de *Vénus* seront également altérés, et la position du centre des deux astres ne sera nullement changée ; la mesure des angles de position devra donc, en théorie, donner des résultats indépendants de toutes les causes d'erreur signalées. Mais on rencontre, dans la pratique, de très-grandes difficultés à obtenir, avec le degré d'exactitude requis par le calcul, une direction fixe servant d'origine aux angles de position. Les expéditions américaines sont, à ma connaissance, les seules qui aient cherché à résoudre le problème ; il est donc nécessaire, pour apprécier la méthode, d'attendre la publication et la discussion de leurs résultats.

2°. *Mesure de la distance des centres.*—La position du centre des deux astres n'est aucunement modifiée, nous l'avons vu, par les phénomènes de diffraction qu'introduit dans l'image un système optique supposé parfait. La mesure de la distance des centres doit donc, théoriquement, inspirer toute confiance, comme celle des angles de position. L'expérience, du reste, semble prouver que les dimensions d'appareils et le mode opératoire des missions françaises peuvent donner la distance des centres avec le degré d'exactitude requis par le calcul.*

Mais il y a, dans ce procédé, une difficulté de même ordre que celle qui consistait, dans le précédent, à déterminer l'origine des angles de position, c'est la nécessité de connaître la *valeur angulaire* des images. Les images focales sont mesurées en unités de longueur ; mais, pour pouvoir comparer entre elles les épreuves obtenues dans les différentes stations, il faut connaître quel angle représente, dans le ciel, un millimètre mesuré sur l'épreuve, ou, ce qui revient au même, sous quel angle on voit, du centre optique de l'objectif, un millimètre placé au foyer. On peut, du reste, déterminer cette valeur de bien des manières différentes :

* On se rappelle que les photographies des missions françaises étaient obtenues directement au foyer d'un objectif d'environ $3^{m}\cdot80$ de foyer. Sur les images directes, sans grossissement, le diamètre du Soleil est d'environ $35^{mm}\cdot84$ et celui de *Vénus* de $1^{mm}\cdot15$. Les images étaient obtenues en outre sur plaques daguerriennes.

la première consiste à mesurer exactement la longueur focale de l'objectif employé (procédé suivi par les Américains) ; on peut encore placer au foyer de l'instrument un micromètre à fils, ou une plaque de verre divisée, et noter le temps qu'une étoile connue met à franchir une distance donnée, quand l'appareil reste fixe. Mais il paraîtra préférable de déterminer la valeur angulaire des images par des procédés purement photographiques. On pourra, par exemple, collimer l'instrument photographique par une lunette méridienne, et photographier ainsi le réticule de cette dernière, dont les fils sont à des distances parfaitement connues. Enfin un procédé employé depuis longtemps par M. Rutherford, et qui lui donne d'excellents résultats, consiste à diriger à former sur la plaque photographique au foyer de l'instrument rigoureusement immobile l'image d'une étoile brillante. Cet astre laisse alors une trace lumineuse continue ; la longueur de cette trace, et le temps pendant lequel elle a été obtenue, permettent de déterminer aisément la valeur angulaire de l'instrument. On peut encore remplacer dans cette méthode l'étoile par le Soleil ; il faut alors, l'appareil restant immobile, prendre deux images du Soleil à un intervalle de temps parfaitement connu, et mesurer la distance des centres de ces deux images. Il est nécessaire, dans ce cas, que les deux images n'empiètent pas l'une sur l'autre ; alors les diamètres du Soleil seraient modifiés dans toute la partie commune aux deux images. J'ai montré, en effet, dans mon premier travail, que les images sont, en général, plus petites lorsqu'elles se forment sur un champ déjà impressionné par la lumière.

Il est, enfin, un dernier procédé qui paraît irréprochable au premier abord, mais qui, dans la pratique, conduirait à des erreurs notables. J'ai montré, en opérant sur des objets uniformément éclairés, à bords rectilignes, et de dimensions très-supérieures à la zone de diffraction, que l'augmentation de l'image d'un objet lumineux est égale à la diminution de celle d'un objet obscur dans les mêmes circonstances. La somme de ces dimensions était constante et égale à ce qu'elle devrait être si l'image se formait seulement d'après les lois de l'optique géométrique (voir la première partie). S'il en était rigoureusement de même pour le Soleil et *Vénus*, les diamètres de ces deux astres, variables en sens inverse d'une épreuve à l'autre, donneraient une somme constante. Cette somme pourrait servir de facteur de réduction pour comparer entre elles les épreuves des différentes stations, et il suffirait dès lors de déterminer la valeur angulaire d'un seul instrument : le rapport des sommes des diamètres de Soleil et de *Vénus*, dans deux instruments, serait celui de leurs valeurs angulaires.

Malheureusement on ne peut compter sur la constance de cette somme, et cela pour plusieurs raisons :

1°. Le diamètre de *Vénus* est loin d'être grand par rapport à l'étendue de la zone diffractée. La théorie montre alors que la diminution du diamètre de *Vénus* n'est pas la même que celle

qui se produirait pour un bord rectiligne ; le Soleil, au contraire, a une courbure assez faible pour pouvoir être considéré comme un bord rectiligne.

2°. L'intensité lumineuse des différents points du Soleil n'est pas du tout uniforme ; elle décroît rapidement vers les bords, par suite de la convexité et de la nature physique de cet astre. Cette décroissance est même tellement accusée que l'intensité photogénique des bords paraît inférieure à celle de la pénombre des taches. Pendant la durée du passage *Vénus* se projettera donc en un point du Soleil beaucoup plus lumineux que n'est le bord, et dont l'intensité varie sans cesse à mesure que *Vénus* se déplace sur le Soleil. L'augmentation du Soleil est alors différente de la diminution de *Vénus*, et la somme de leurs diamètres est variable avec toutes les conditions de l'expérience, sensibilité de la plaque, durée de pose, transparence de l'atmosphère au moment ou l'on fait la photographie, etc.

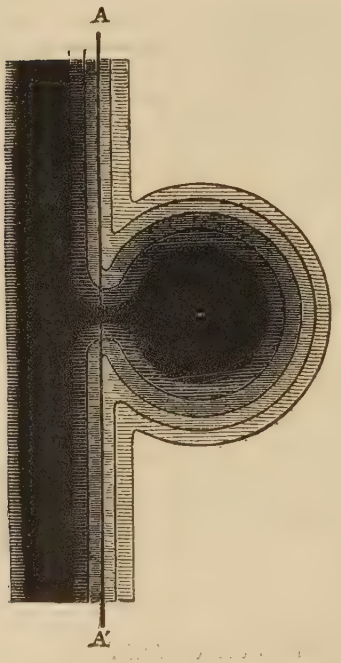
Toutes ces prévisions ont été confirmées par l'expérience directe à l'appareil à passages artificiels, et, mieux que tout le reste, la mesure des épreuves du passage même de *Vénus*, obtenues dans tous les pays et dans les circonstances les plus variées, doit mettre ces causes d'erreur en évidence et montrer que, pour déterminer la valeur angulaire des instruments, il faut avoir recours à l'une des méthodes que j'ai exposées plus haut, en renonçant complètement à la constance présumée de la somme des diamètres des deux astres.

II. *Détermination de l'Instant des Contacts.*

La détermination de l'instant des contacts, qui peut être tentée par la photographie pour contrôler l'observation directe s'obtient de deux façons différents, soit par l'observation du contact même, soit interne, soit externe, des deux disques, soit par la mesure de la corde commune à *Vénus* et au Soleil au moment où la planète entre sur le bord de l'astre. En étudiant comment varie, avec le temps, cette *distance des cornes* ou corde commune, on en déduit par extrapolation le moment où cette distance est nulle, c'est-à-dire le contact. Comme les causes d'erreur qui s'attachent à ces deux procédés sont les mêmes nous les réunirons pour la discussion.

La théorie de la diffraction permet de calculer toutes les phases du passage de la planète sur le bord du Soleil dans un cas simple, celui où l'on suppose que *Vénus* est formée par un disque ou par une sphère opaque sans atmosphère, et que le Soleil a une intensité lumineuse constante en chacun de ses points. On trouve de la sorte que, dans l'intérieur même de la partie géométrique de l'image, l'intensité décroît quand on s'approche des bords, et forme une zone estompée qui s'étend bien en dehors de l'image

géométrique. On peut ainsi obtenir par points une série de *courbes de niveau*, pour ainsi dire, tout le long desquelles l'intensité lumineuse est constante. La figure ci-jointe représente grossièrement la forme générale de ces courbes au moment même du contact géométrique interne. La courbe AA′, tout le long de laquelle l'intensité est la moitié de ce qu'elle est au centre du Soleil, se confond avec le bord géométrique de cet astre pour tous les points un peu éloignés du point de contact; dans les régions de *Vénus* distantes du même point, la courbe est concentrique à l'image géométrique et un peu intérieure (elle ne coïncide plus avec le bord géométrique à cause de la courbure de *Vénus*); enfin, aux environs du point de contact, ces deux courbes se raccordent de façon à reproduire exactement l'apparence désignée sous le nom de goutte noire ou de ligament. Cette apparence est exagérée encore dans les courbes d'intensité $\frac{1}{4}$ et $\frac{1}{8}$ que représente également la figure. On obtient des traces analogues pour chaque instant du passage.



D'autre part, une plaque photographique, exposée à la lumière pendant un temps déterminé, donne une image qui comprend tous les points dont l'intensité lumineuse est supérieure à une certaine limite. L'impression s'arrête à une de ces courbes de niveau que j'ai indiquées plus haut, et comprend des courbes de niveau de plus en plus faibles, à mesure que l'on augmente la sensibilité de la plaque, l'intensité de la lumière, et la durée de pose.

Ce sont justement les conditions dans lesquelles ont été obtenues mes photographies. L'appareil à passages artificiels était amené à une position convenable, puis fixé, et on faisait sur la même plaque cinq épreuves successives, correspondant à des durées de pose progressivement croissantes. On obtient ainsi la forme de cinq courbes de niveau de plus en plus faibles, et il est frappant de voir avec quelle fidélité l'expérience réalise les formes indiquées par la théorie. La même expérience a été répétée en donnant à la planète les positions qu'elle occupe de dix en dix, ou de quinze en quinze, secondes pendant les cinq minutes

qui précèdent le contact interne d'entrée et pendant les deux minutes qui le suivent.

Des résultats analogues avaient déjà été partiellement, du reste, obtenus à Greenwich et par M. Janssen, dans les expériences préliminaires faites avant le départ des expéditions pour le passage de *Vénus*: on avait quelquefois observé des apparences de goutte noire quand l'appareil à passages artificiels était éclairé très-fortement par la lumière directe du Soleil, par exemple.

Reste à déterminer l'erreur que ces phénomènes peuvent introduire dans l'observation photographique de l'instant des contacts, surtout des contacts internes. Or, la théorie montre que l'apparence générale ne change pas brusquement au moment des contacts. Même alors que la planète est entièrement entrée sur le Soleil, l'apparence de la goutte noire persiste tant que le filet lumineux compris entre *Vénus* et le bord du Soleil n'a pas une dimension égale ou supérieure à celle de la zone de lumière diffractée. Il sera donc possible d'obtenir encore une *goutte noire* après que le contact géométrique aura eu lieu : il suffira, pour cela, de poser très-peu de temps.

La photographie a, dans cette observation, un désavantage considérable sur l'observation optique directe. L'œil, en effet, à côté d'une lumière très-vive, comme celle du Soleil, devient incapable de percevoir des intensités assez faibles : la perception optique s'arrêtera donc généralement à des courbes de niveau peu distantes du bord géométrique, tandis qu'en augmentant la sensibilité des plaques photographiques ou la durée de pose, on ira chercher des courbes d'intensité aussi faible que l'on voudra. Dans l'observation du passage, même avec des plaques identiques et une durée de pose constante, il suffira du moindre changement dans la transparence de l'atmosphère pour que l'impression photographique s'arrête à des courbes de niveau différentes, et que de deux épreuves successives la dernière semble correspondre à une période antérieure du phénomène. Les effets indiqués diminuent si l'on augmente l'ouverture de la lunette, mais ils croissent considérablement pour peu que l'objectif employé possède la moindre aberration. Avec un objectif non rigoureusement aplanétique, j'ai pu obtenir encore un contact apparent par des durées de pose très-faibles, alors que *Vénus* était complètement entrée sur le Soleil depuis *une minute* de temps.

Quant aux contacts externes, la diffraction modifie la forme de l'échancrure que *Vénus* produit sur le Soleil ; mais, comme il s'agit seulement de constater la présence de cette échancrure, il est possible que l'observation photographique donne des résultats de même valeur que l'observation optique.

Reste un dernier moyen, qui consiste à étudier comment varie, avec le temps, la corde commune à *Vénus* et au Soleil, pendant l'entrée ou la sortie, et à en déduire par extrapolation le moment où cette corde est nulle, c'est-à-dire les contacts. Ce

procédé présente, en photographie, des difficultés particulières, puisque pour un même instant la longueur de la corde peut varier entre des limites considérables avec la sensibilité de la plaque, la durée de pose, ou la transparence de l'atmosphère.

Toutes ces conclusions, vérifiées directement par l'expérience, sont déduites de l'hypothèse d'un planète sans atmosphère, et de l'égale intensité lumineuse des différents points du Soleil; mais on doit reconnaître que le phénomène réel est encore plus complexe. Nous espérons pouvoir, M. André et moi, déterminer expérimentalement les modifications que la présence d'une atmosphère autour de *Vénus* amène dans les résultats auxquels nous conduisent nos premières expériences.

En résumé, l'observation photographique du passage de *Vénus* présente des difficultés au moins aussi grandes que l'observation optique; mais on peut conserver tout espoir d'arriver par ce moyen à une bonne détermination de la parallaxe, surtout si l'on emploie les méthodes de mesure de distances de centres et d'angles de position, méthodes propres à la photographie. En tous cas, le prochain passage de *Mercure* (Mai 1878) devra être observé avec le plus grand soin, afin de vérifier d'abord sur un phénomène céleste nos conclusions, basées sur des expériences de passages artificiels, et d'assigner ensuite exactement les limites entre lesquelles les causes d'erreur signalées peuvent varier dans la nature.

A Determination of the Semi-diameter of Venus at the mean distance of the Sun from the Earth. By A. W. Downing, B.A.

In this investigation I have made use of the results of the Washington Transit Circle Observations during the years 1866–1872 inclusive; the Transit Circle having been brought into use in 1866, and the results of 1872 being the most recent to which I have access. During this interval the value of the semi-diameter of *Venus* at the mean distance of the Sun from the Earth, given in the American Ephemeris, and with which the Washington Observations have been compared, was $8''{\cdot}546$.

Let the true semi-diameter $= 8''{\cdot}546\ (1+y)$, so that $y =$ coefficient of variable part of correction depending on the planet's distance; and let $x =$ constant part of correction arising from irradiation &c.; we have then a series of equations of the form

$$x + y \times \text{Tab. Semi-diameter} = \text{Correction to Ephemeris.}$$

Using all the observations of vertical diameter of *Venus* made during the above-mentioned years, I have formed 167

equations in this manner; solving these by the method of least squares, I obtain the equations—

$$167x + 1636{\cdot}3y = 40{\cdot}3$$
$$1636{\cdot}3x + 23565{\cdot}24y = 239{\cdot}11;$$

whence

$$x = 0{\cdot}4439 \pm 0{\cdot}1036 \times e,$$

where e is the probable error of an observation, and

$$y = -0{\cdot}02068 \pm 0{\cdot}00777 \times e.$$

Therefore

$$\text{True Semi-diameter} = 8''{\cdot}546\,(1+y) = 8''{\cdot}3693 \pm 0{\cdot}0664 \times e.$$

The value $8''{\cdot}546$ was determined by Prof. Peirce from 30 observations made with the Washington Mural Circle during the year 1846.

22 *Waterloo Road, Dublin,*
1877, *May* 9.

Second Catalogue of Micrometrical Measurements of Double Stars made at the Temple Observatory. By J. M. Wilson, M.A., and G. M. Seabroke, Esq. (Abstract.)

This Catalogue contains the measures made at the Temple Observatory during the years 1875 and 1876, and some few measures made in previous years that were omitted in our first Catalogue.

Most of the measures were made by Mr. Seabroke, Mr. Wilson having been unable to work at the Observatory during 1875 and part of 1876. Mr. Seabroke's measures were, however, frequently confirmed by those of Mr. Percy Smith. They were all made with the parallel wire micrometer, and a magnifying power of about 400 on the Alvan Clark refractor of $8\frac{1}{4}$ inches.

The Temple Observatory is now being built in a permanent form, with a Curator's house adjoining, and we take this opportunity of a break in our series of measurements to offer this second Catalogue to the Society, in the belief that double-star measures are more valuable when published at frequent intervals.

We thought it unnecessary to print the R.A. and Dec. of Struve's stars. All observers who are likely to use this catalogue will have them at hand.

Lunar Eclipse of the 27th February, 1877, observed at Cadiz.

By Augusto T. Arcimis.

I have long delayed reporting my observations of this eclipse because I even doubted the reality of what I had seen, but having read in the scientific journals that the phenomenon had everywhere presented a similar aspect, I venture to offer the result of my observations, which can no longer be received with doubt or mistrust.

The Sun set at $5^h\ 35^m$. The clouds which gathered in the east prevented my seeing the Moon until $6^h\ 20^m$; it was already eclipsed, and in spite of the crepuscular light, the characteristic copper tint was, though faintly, visible in the centre of the disk, the other portions being covered by the clouds.

At $6^h\ 46^m$ the Moon is red throughout to the naked eye and presents the appearance, were that possible, of an annular eclipse; the centre is dark and not a single detail of its surface is distinguishable; it is surrounded by a luminous fiery corona. This strange aspect disappears through the telescope, and the eclipse presents the normal appearance.

The clouds now and then interrupt the observation.

At 7^h the red ring breaks towards the south, and the inroad of the shadow becomes distinctly visible to the naked eye. Through the telescope only the western edge, surrounded by the penumbra, can be distinguished, while the rest of the disk presents a dull grey colour so deep that it effaces every detail of the lunar surface.

The clouds again invade the sky.

At $7^h\ 41^m$ it clears partially, and the crater *Grimaldi* emerges from the shadow.

At $7^h\ 49^m$ emersion of *Kepler*.

At $7^h\ 57^m$ „ „ *Kopernicus*.

At $8^h\ 0^m$ the shadow crosses the centre of *Plato*.

At $8^h\ 1^m$ emersion of *Tycho*.

At $8^h\ 5^m$ „ „ *Archimedes*.

At $8^h\ 7^m$ „ „ *Ptolemeus*.

At $8^h\ 11^m$ „ „ *Manilius*.

At $8^h\ 15^m$ „ „ *Menelaus*.

At $8^h\ 21^m$ the line of the shadow reaches the crater *Endymion* and the edge of *Mare Australe*.

At $8^h\ 28^m$ the shadow touches the edge of *Mare Crisium*.

At $8^h\ 36^m$ the last contact takes place at 90° from the boreal pole towards the west.

This eclipse presented a notable aspect, and the corona of red light which spread over the internal edge of the Moon elicited the admiration even of strangers to the science. An unforeseen circumstance prevented my using the spectroscope.

A Table for facilitating the determination of Mean Equinoctial Time to the end of the present Century.

By Vice-Admiral Sir Charles Shadwell.

In the *Nautical Almanac*, from the year 1834 (the date of its first appearance in its present improved form) up to 1875 inclusive, information was formerly given, at page 20 of each month, to facilitate the determination of 'Mean Equinoctial Time.'

From and after 1876 this information has been altogether discontinued. No reason has been assigned, either in the 'Annual Prefaces' or in the 'Explanations' of that volume, for this omission. It is, however, understood to have been withdrawn because it had never come into general use among astronomers, as had been anticipated at the time of its first introduction.

Since the use of equinoctial time afforded a means of reckoning time free from the necessity of specifying any particular geographical locality, being measured from an epoch common to all the world (viz. the passage of the mean Sun through the mean vernal Equinox) and identifiable independently of all localities, thus avoiding all ambiguities in its description, its disappearance from the pages of the National Ephemeris is, I think, to be regretted.

With a view to rectify this omission to a certain extent, and to afford those desirous of employing equinoctial time the means of doing so in future, the accompanying Table, giving the data for its determination up to the end of the present century, is herewith submitted to the notice of the Royal Astronomical Society :—

Table for facilitating the determination of Mean Equinoctial Time.

Equinoctial Year.	Date of Commencement. March.	Number. N.	Fraction of Equinoctial Time for the Year to be added to the Equinoctial Day. d
1875–76	22	81	0·574169
1876–77	21	81—80	0·331953
1877–78	21	80	0·089737
1878–79	22	81	0·847521
1879–80	22	81	0·605305
1880–81	21	81—80	0·363089
1881–82	21	80	0·120873
1882–83	22	81	0·878657
1883–84	22	81	0·636441
1884–85	21	81—80	0·394225

Equinoctial Year.	Date of Commencement.	Number. N.	Fraction of Equinoctial Time for the Year to be added to the Equinoctial Day.
	March.		d
1885–86	21	80	0·152009
1886–87	22	81	0·909793
1887–88	22	81	0·667577
1888–89	21	81—80	0·425361
1889–90	21	80	0·183145
1890–91	22	81	0·940929
1891–92	22	81	0·698713
1892–93	21	81—80	0·456497
1893–94	21	80	0·214281
1894–95	22	81	0·972065
1895–96	22	81	0·729849
1896–97	21	81—80	0·487633
1897–98	21	80	0·245417
1898–99	21	80	0·003201
1899–1900	22	81	0·760985

Explanation of the Table.

This Table contains for the equinoctial years named, that is for the years 1875–76 to 1899–1900 inclusive, 1st, in column 2, the day on which the equinoctial year commences; 2ndly, in column 3, the number N; 3rdly, in column 4, the fraction of equinoctial time to be added to the day of the equinoctial year.

N is the number of days which is to be subtracted from the 'day of the year' (increased if necessary by 365) to give the corresponding day of the equinoctial year; the day of the year corresponding to any given date being taken from the Table published annually in the *Nautical Almanac* commencing with 1876 ('Day of the Year,' &c., pp. 484 &c.)

This number is generally 81, but sometimes 80. When both numbers are given, 81 is to be subtracted from the day of the year between the vernal Equinox and December 31, and 80 between January 1 of the following year and the succeeding Equinox.

Application of the Table.

I. To find when the equinoctial year commences:—

To the date in column 2 add the complement to unity of the fraction in column 4.

Thus the equinoctial year 1884–85 commences on March $21^d + (1 - 0{\cdot}394525)$, or on March $21^d{\cdot}605775$ Greenwich Mean Time.

So again the equinoctial year 1894–95 commences on March $22^d+(1-0^d{\cdot}972065)$, or on March $22^d{\cdot}027935$ Greenwich Mean Time.

II. To find the mean equinoctial time corresponding to Greenwich mean noon on any given day:—

From the day of the year, increased if necessary by 365, subtract the number N, and affix the fraction for the year from column 4.

Thus: Required the equinoctial time at Greenwich mean noon on May 13, 1878.

Here the day of the year corresponding to May 13 would be 132, and in the equinoctial year 1878–79 N is 81, and the fraction of equinoctial time $0^d{\cdot}847521$.

Therefore, $132^d-81^d+0^d{\cdot}847521$; or $1878^y\ 51^d{\cdot}847521$ is the mean equinoctial time at Greenwich mean noon on the day in question.

Again: What will be the equinoctial time at Greenwich mean noon on February 5, 1893?

The day of the year corresponding to February 5, 1893, is 35, and in the equinoctial year 1892–93, N between January 1 and the following vernal Equinox is 80, while the fraction of equinoctial time to be added is $0^d{\cdot}456497$.

Hence, $35^d+365^d-80^d+0^d{\cdot}456497$, or $320^d{\cdot}456497$, is the mean equinoctial time at Greenwich mean noon on February 5, 1893, which is required.

Note.—Should it ever be deemed expedient to reintroduce "mean equinoctial time" to a place in the *Nautical Almanac*, it would not be necessary to assign to it a column in a page for each month, as under former arrangements previous to 1876. All that would be requisite would be to give, in a note at the foot of the Table "Day of the Year, &c.," the number N and the fraction of equinoctial time to be employed, both before and after the vernal Equinox of the current year, with a brief explanation of how they were to be applied; this last being given in the "Explanation of the Articles" at the end of the work.

A very slight alteration of the present arrangement of the type in the Table referred to, and a corresponding brief addition to the "Explanation," would permit this.

Meadowbank, Melksham,
1877, *April.*

Description of an Improved Diagram for the Graphical Solution of Spherical Triangles, applicable to the questions arising out of the Spheroidal Figure of the Earth, treated in the Paper read before the Society Nov. 10 ult., and further illustrated by the case of the Prediction of Occultations. By F. C. Penrose, Esq.

The Table, which should have accompanied the paper referred to, giving the values of n—namely the distance between the centre of the Earth and the foot of the normal in different latitudes—and

for the correction of the Tabular Parallax when referred to the same point, is as follows. The Horizontal Equatoreal Parallax is taken as equal 57′. When more or less than this, the figures below should be adjusted proportionably :—

Latitude.	Correction to Polar Distance. Subtract from P.D.	Correction to the Equatoreal H.P. Add to H.P.
	"	"
0	0	0
5	2·01	0·10
10	4·01	0·35
15	5·97	0·80
20	7·89	1·34
25	9·74	2·06
30	11·52	2·88
35	13·22	3·80
40	14·81	4·75
45	16·29	5·76
50	17·64	6·76
55	18·85	7·72
60	19·93	8·60
65	20·85	9·40
70	21·61	10·15
75	22·21	10·70
80	22·65	11·15
85	22·90	11·40
90	22·99	11·49

The diagram attached to the paper referred to, although it answers its purpose sufficiently well for working the spheroidal corrections, is not so well adapted for the general solution of a spherical triangle, because the lines representing the different zenith distances diverge so much from one another that it becomes troublesome to interpolate, and it offers but little facility for the converse problem of obtaining altitudes from given hour angles and polar distances; moreover, when the angle at the Moon is small, it does not suffice for determining the angle with sufficient accuracy.

The new diagram avoids these imperfections. The arrangement of the lines due to the different zenith distances is much more equable, so that interpolations can be effected with tolerable certainty.

This also applies to the interpolations required for using a diagram prepared for a particular latitude for latitudes within a few degrees of it.

If diagrams similar to the one submitted were made for every fifth degree of latitude, they would suffice, as I find by trial,

within limits of error of about 30′, to determine any zenith distance from a given hour angle and polar distance, or for performing the converse operation of finding the hour angle or polar distance from the other two given data: and with more accuracy, of course, if a larger scale were adopted. I exclude, of course, unfavourable cases; such, for instance, as are produced by ill-conditioned triangles, which would even be avoided when logarithmic calculation is used.

This diagram belongs to latitude 51° 30′, and is, in fact, a projection of a sphere on a method not much unlike that of Mercator. It gives the relation between the three sides of a spherical triangle and one of the angles. Two of the sides and the angle may vary indefinitely, but one side is constant for the particular diagram. The angle referred to is adjacent to the constant side. For instance, if the constant side be supposed to be the co-latitude, the vertical measurements give polar distances and the oblique curved lines give zenith distances; but for general use I speak of the latter as the base, and the former as the side, and of the angle as the included angle—that is, included between the constant and the variable side. For instance, it will be seen by inspection of the diagram, that if we combine the vertical line drawn downwards from VI^h or 90° on the upper horizontal row of figures with 50° drawn across from the vertical column on the left, they will meet very near to the oblique line representing the base 60°. This, using the notation given above, is stated thus:—

The side 50° and included angle 90° have approximately for their base 60°. (This is true for 59° 48′.)

In a lunar triangle, we may thus obtain approximately the zenith distances of the Moon and star, and the parallactic angles of the two bodies. For this purpose generally two diagrams will be necessary, one adapted to the latitude and the other to the lunar distance. The following case, however, has been chosen as adapted to the one diagram herewith submitted.

In the figure below, if PSM, PZM be supposed to represent the measurements requisite for a lunar distance observation, the observer may be supposed to know

PZ	The co-latitude;
PM	The Moon's polar distance, by computation;
PS	The star's „ „
SM	The lunar distance, by measurement;
ZPM	The Moon's hour angle, by computation;
ZPS	The star's „ „

What are required are the two parallactic angles, viz. ZSM and ZMS, and the sides ZM and ZS, and also the angle PMS. The two sides indeed may, under favourable circumstances, be observed; but it is often impossible or inconvenient to do so. The diagrams enable all these quantities to be obtained by inspection with sufficient accuracy for general purposes, and,

should more exactness be required, the solution by this means offers a valuable check.

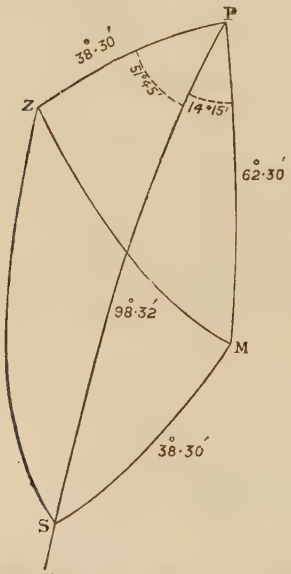
Thus all the data required for clearing the lunar distance may be determined.

Should the azimuth be required, it may be got, *mutatis mutandis*, as readily as the parallactic angle.

The simplest way of disposing of the spheroidal correction is by at once referring both the observation and the lunar triangle to the foot of the normal. When this is done, the geocentric lunar distance given in the three-hourly Arguments of the *Nautical Almanac* is, for any other latitude than the equator, too large or too small according as the angle PMS is acute or obtuse, and the excess or defect $= n \cos D \cos M$, where n is the quantity given above in the Table for the correction of the Moon's declination, and M is the angle PMS.

It seems, however, on several accounts more convenient to make this adjustment upon the measured distance, instead of the tabular argument, but in the contrary direction, so that when M is acute the distance must be increased, and decreased when it is obtuse, by the amount above stated.

As respects the increment to the parallax given in the Table, it should receive its due proportion when the parallax in altitude is computed.



As an example of reducing a lunar distance with the help of the diagram, let it be supposed that the altitudes have not been measured, but the distance only.

Then, in the lunar triangles PZM, ZMS, after computing the hour angles and polar distances of the Moon and star, we have—let us suppose—

		°	′
PZ	=	38	30
PM	=	62	30
SM	=	38	30
PS	=	98	32
SPM	=	14	15
SPZ	=	51	45

In the spherical triangle PMS the use that is made of the given side SM is in the selection of the proper diagram, and the same applies to the use of the side PZ in the spherical triangle PZS.

Then, combining 62° 30′ as a side with 98° 32′ as a base, we obtain the included angle 157°. This is the angle PMS.

Then, in triangle PZS, using 98° 32′ as a side and 51° 45′ as an included angle, we obtain for the base PS 74° 30′.

Then, in triangle PZM, using 62° 30′ as a side and ZPS + SPM = 66° as an included angle, we get for the base 54°.

Then, in triangle ZSM, with 74° 30′ as a side and 54° as a base, we obtain the star's parallactic angle ZSM = 50° 30′; and lastly, in the same triangle, using 54° as a side and 74° 30′ as a base, we obtain ZMS, the Moon's parallactic angle = 112°.

The corrections may now be made.

	″
The effect of the increased value of parallax in altitude = 7″ sin 54° cos 112°, increasing the distance by .	2·1
But the distance must be decreased, on account of the term n cos D cos PMS, by	15·3
Total correction	13·2

This example is not an imaginary case, but would have occurred in latitude 51° 30′ S. and longitude 4^h 34^m W. on Feb. 9th ult. in the distance of α *Aquilæ* given in the *Nautical Almanac*. As the parallax then was 54′, the amount above given must be reduced to 12″·6. But this quantity would have affected the longitude by 52^s, and by more than 8 nautical miles in distance. And this is by no means an extreme case.

I showed in the former paper how this approximate determination of the altitudes and angles, &c., might serve to facilitate the accurate clearing of the lunar distance. I will proceed to show how it can be used to assist in predicting occultations and eclipses. At the bottom of the diagram is added a scale of logarithmic sines similar to that used on the slide rule, which will enable opposite sides or angles to be ascertained without recourse to another diagram.

As an example of its use, take the figure already given, and in the triangle PZM let it be required to find the angle PMZ. Since log sin PMZ − log sin PZ = log sin ZPM − log sin ZM, the difference on the logarithmic scale between PMZ and 38° 30′ must equal the distance between ZPM and ZM. Measure then the interval between 66° and 54° on this scale and apply it to 38° 30′, and it will be found to reach to 45°, which is very nearly the true result. It should be observed that when the angles approach 90° this scale is not suited for accurate determination. At such points, however, the diagrams are at their maximum of efficiency.

For the purpose of predicting occultations, an additional diagram is prepared with certain requisite scales, especially one of minutes of arc for different values of parallax. It is suitable to all latitudes.

Having prepared the proper elements, lay down the Moon's course on the occultation diagram, and, with the hour angle and polar distance, find, on a diagram of zenith distances suited to the latitude, the zenith distance and parallactic angle at the moment of conjunction: the former is required for ascertaining the amount of the Moon's parallax in altitude, and the latter for its direction with reference to the point C, which represents the

centre of the earth, or rather the foot of the normal belonging to the latitude.

When the position of the place for which the prediction is made is laid down for the moment of conjunction, it is seldom difficult to decide in what part of the Moon's path the phenomenon is to be looked for *within an hour*. Make, then, a new computation of the parallax in altitude and the parallactic angle, and lay down and join the points which have been determined. A scale is given for dividing any hourly segment of the Moon's path into 30 subdivisions, and there is little difficulty in dividing a corresponding hour interval on the line joining the two places of the terrestrial observatory [a segment always of an elongated ellipse, and which, it may be observed, is generally concave for north and convex for south declinations] into intervals representing 10^m each. It is now easy to see which of the corresponding intervals are separated most nearly to the extent of the Moon's semidiameter, and what minute of time offers the best prediction. An example is given, namely, a prediction of the occultation of ρ *Leonis* Jan. 30, 1877, for Greenwich.

Elements of Occultation. ρ *Leonis*, Jan. 30, 1877.

Prediction for Greenwich.

		☌'s R.A.	☌'s Dec. approx.	Geocentric diff.
	h m s	h m s	° ′	′ ″
G.M.T. of ☌	12 5 11	10 26 22	9 56	27 36
	0 1 59			−0 18 to normal.
	20 39 18			27 18
G.S.T. of ☌	8 46 28			

Hour angle $1^h\ 40^m$ Observer west of Moon.

Parallax.

	′ ″
geocentric	60 52
	+ 0 7
	60 59

	′	″
Moon's Motion in R.A.	34	10
Eastwards	33	50
And in Decl. southwards	16	8

With these measures of the Moon's motion lay down the Moon's course upon the occultation diagram, and HO will be the Moon's course during the hour preceding conjunction; and with the hour angle and declination above given obtain from the general latitude diagram, using them as an included angle and side, the zenith distance at conjunction.

	°	′
This appears to be	46	30
The logarithmic sine scale will then give the parallactic angle, about	21	30

The parallax 60′ 59″ set off upon a radius drawn at an angle of 46° 30′ from CZ will reach to *a*.

The horizontal distance from a to ZC will be the parallax in altitude.

Set this off as CA, making an angle of 21° 30′ with CZ.

A is the observer's place at conjunction.

It is at once evident that the Moon's disappearance must have taken place more than an hour before conjunction. Take, then, an hour angle of $3^h\ 10^m$ or $1\frac{1}{2}$ hours before ☌, and with this as an included angle, and with the star's polar distance as a side, rework the observer's place. This will now be found to have

Zenith distance about 56° 30

and

Parallactic angle . 33 35

With these measures determine Cb and CB in the same manner as Ca and CA were done before. B is then the observer's place $1\frac{1}{2}$ hours before conjunction. Join AB—remembering that it is a portion of an elongated ellipse. Measure backwards upon the Moon's course Hh = $\frac{1}{2}$ HO, and h becomes the Moon's place corresponding to B. By means of the diverging scales on the drawing, the line AB may be divided into 9 divisions each measuring 10 minutes, and Hh into 15 divisions each measuring 2 minutes. Having marked off as many of these divisions as may seem requisite, take the Moon's semidiameter suited to the parallax 60′, and it will be found to reach across and bridge the interval between the two points on the lines Hh and AB due to the time $1^h\ 14^m$ approximately before conjunction. By working in a similar manner the interval for the reappearance, it might be shown to agree most nearly with the time 8 minutes before conjunction;

These Greenwich times therefore are 10 51

and 11 57

These predictions are identical with those given in the *Nautical Almanac*, but are quite independent of it.

If the angle at the Moon's vertex for the observation is required, join the centre C with the observer's place at the time of the phase in question, and having drawn a circle representing the Moon, draw a straight line through its centre parallel to the former one. It will cut the Moon's circumference at the proper points for estimating the angle required. The figure gives the erect image, and must be allowed for accordingly when required for the inverted image.

On a Method of Destroying the Vibrations in a Mercurial Reflector.

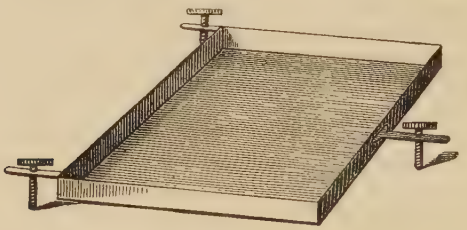
By M. de Boë, of Antwerp.

(*Communicated by Capt. Wm. Noble.*)

M. de Boë (Anvers) pense avoir perfectionné le système d'observation des fils de la lunette méridienne sur le bain de mercure.

Il emploie une cuvette à trois vis calantes, au fond de laquelle il place une plaque daguerrienne, sur laquelle il laisse tomber quelques gouttes de mercure. A l'aide de ce procédé les vibrations sont à peu près complètement annulées.

La cuvette est à fond plan.



Avant de laisser tomber le mercure sur la plaque daguerrienne, on la frotte avec le doigt et un peu de mercure ; alors le fond du liquide devient plus adhérent, ce qui est la principale cause qui empêche les vibrations.

It will be seen that the method of M. de Boë is merely a modification of that described by Prof. Pritchard before the Roya Astronomical Society. (*Monthly Notices*, vol. xiii., Jan. 1853, pp. 61-66; see also vol. xxxvi., Dec. 1875.)—W. N.

On a Method of making a Pendulum swing in an approximately Cycloidal Arc. By General M. C. Meigs, U.S.A.

I have been amusing myself by making a "Baily compensating seconds pendulum" for an old clock.

On hanging it for the first time I notice that it twists in swinging. The twist is synchronous with the vibration of the pendulum, and at the middle of the vibration, when the pendulum is vertical, its suspending spring is flat and straight. The twisted spring is shorter than the flat one, and this twisting therefore shortens the radius of curvature of the arc described by the centre of oscillation after it passes the vertical on each side.

By suitably proportioning the width and thickness of the suspending spring, this shortening of the radius might be so regulated that the arc described would be very nearly, and if the work is perfect accurately, cycloidal.

The twist may be given by making the crutch strike a round bar placed in front of the pendulum rod, and not in the plane of vibration: the further this bar which receives the impulse is from the plane of the pendulum, the greater the twist.

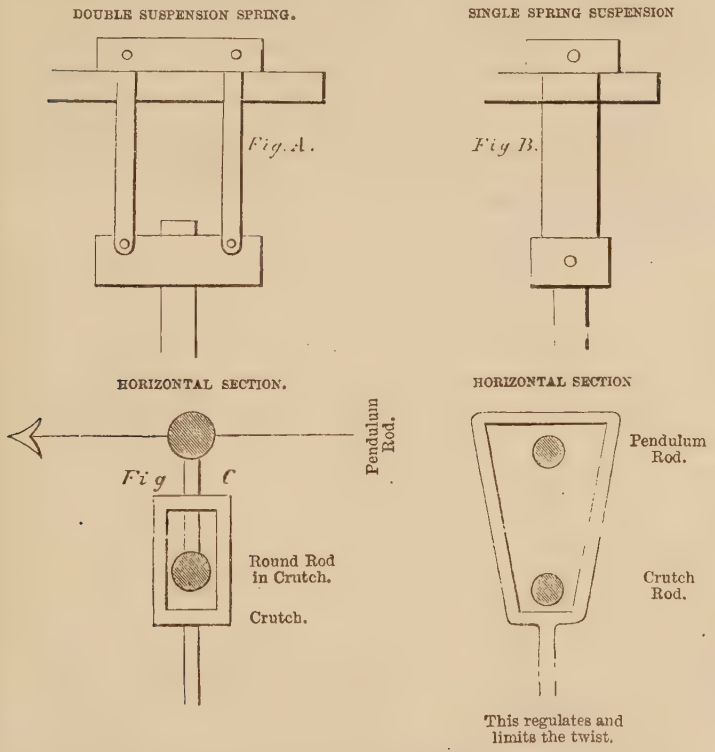
There are, then, two means of regulating the curvature of the arc, viz. :—

1. Width and length of suspension spring, or, when two parallel springs are used, the distance between them.

2. Distance from the bar which receives the impulse, to the plane of the pendulum.

I have never seen these arrangements noted in any publication on the pendulum, and if not heretofore tried, I think it might be well to try them. Huygens' cycloidal cheeks failed to secure superior accuracy.

The ordinary suspension spring probably causes the pendulum to swing in an approximately cycloidal arc, yet it is not generally exact. I think this twisting motion can be made to produce an absolutely isochronous motion through the greatest ordinary or necessary range of change in the arc of vibration of the pendulum of the astronomical clock.



These sketches illustrate constructions suggested. There is a wide range in the disposition of A, B, and C.

Quartermaster-General's Office,
Washington, April 12, 1877.

H H

On the Opposition of the Minor Planet Melpomene as a means of determining the Solar Parallax. By Mr. David Gill.

The accompanying list contains the Stars of Comparison to be observed with *Melpomene*. The same remarks apply to them as to the case of *Mars* and *Ariadne*, for which the comparison stars were published in the last number of the *Monthly Notices*.

In the case of *Melpomene* the necessity for Meridian observations of the comparison stars is specially urgent, because from lack of suitable stars a very rigid system of heliometer triangulation cannot be arranged.

I am indebted to Dr. Auwers of Berlin for the daily places of the planet, by which I have been enabled to select the stars of comparison.

I had also intended to observe the planet *Iris*, but this could not be done advantageously without taking up a new station in about 20° N. latitude. The circumstances of the opposition of *Melpomene* are so little less favourable than *Iris* that the risk and loss of time incurred appeared to more than counterbalance the advantages offered by the somewhat nearer approach of *Iris*.

The circumstances of the opposition of *Iris*, are, however, very favourable for the application of Dr. Galle's method, and it is to be hoped the opportunity will not be lost.

Stars to be observed with Melpomene at Opposition, 1877.

Adopted Name of Star.	Approximate. 1877·0 R.A. h m s	Dec. ° ′	Mag. in Berlin Academy Star Cat. Mag.	No. in Weisse's Bessel.	Mag.	No. in Schjellerup.	Mag.	No. in Lalande.	Mag.	Remarks.
a	4 17 16	+2 45	8	iv. 326	8	...	...	8266	8½	
β	4 18 1	2 12	8	340	8	1396 1397	8·5 8			
γ	4 19 39	0 46	7–8	381	8	...	...	8348 8349	7 8	
b	4 22 26	2 6	8–9	...	...	...	...	8437	8	
δ	4 27 14	0 46	8	544	8	...	...	8601	8½	
c	4 27 18	3 11	9	...	...	...	...	...	...	617 Arg. Zones +3°
d	4 29 24	1 11	8–9	...	...	...	...	...	...	777 „ „ +1°
e	4 30 38	2 20	9	...	...	...	...	...	...	737 „ „ +2°
f	4 32 46	1 54	8–9	685	8–9					
g	4 34 33	2 17	8	727	8					
y	4 34 39	0 44	8	...	...	...	...	8824	8	
h	4 35 8	+1 40	...	...	...	1499	8·5	...	...	798 Arg. Zones +1°

Adopted Name of Star.	Approximate. 1877·0 R.A. h m s	Dec. ° ′	Mag. in Berlin Academy Star Cat. Mag.	No. in Weisse's Bessel.	Mag.	No. in Schjellerup.	Mag.	No. in Lalande.	Mag.	Remarks.
i	4 36 33	+2 11	8–9	774	8–9	...	...	8883	8½	
ϵ	4 36 40	0 52	8–9	777	8–9	1512	8·7			
k	4 38 54	2 25	8–9	817	8–9					
l	4 43 9	2 0	8	902	8	...	...	9066	8	
m	4 44 26	0 56	7	936	7	...	...	9101	7	
x	4 45 51	1 38	...	...	...	1564	8·3	...	...	837 Arg. Zones +1°
n	4 49 46	2 59	8	1067	8	1581	8·5	9268	7	
o	4 49 48	1 51	8	...	...	...	...	9270	8	
r	4 53 39	3 6	8–9	...	...	...	...	9400		
s	4 56 3	1 52	8–9	1223	8–9					
t	4 58 35	+2 29	8–9	...	...	...	...	9561	8½	

Plan of Heliometric Triangulation of Stars to be compared with Melpomene.

Stars.	Mag.	A.R. of Middle Point. h m s	Dec. of Middle Point. ° ′	Position Angle. °	Dist. ′
a—*β*	8—8	4 17 38	+2 29	341	35
a—*b*	8—8-9	19 51	2 26	297	87
β—*b*	8—8-9	20 14	2 9	275	66
β—*γ*	8—7-8	18 50	1 29	344	89
γ—*b*	7-8—8-9	21 2	1 26	208	90
γ—*δ*	7-8—8	23 27	0 46	270	114
b—*c*	8-9—9	24 52	2 39	228	98
b—*d*	8-9—8-9	25 55	1 39	298	118
b—*δ*	8-9—8	24 50	1 26	318	108
δ—*d*	8—8-9	28 19	0 59	232	41
δ—*e*	8—9	28 56	1 33	208	107
δ—*f*	8—8-9	30 0	1 20	231	107
δ—*y*	8—8	30 57	0 45	271	111
d—*e*	8-9—9	30 1	1 46	195	71
d—*f*	8-9—8-9	31 5	1 33	230	66
d—*g*	8-9—8	31 59	1 44	230	102
d—*h*	8-9—8-9	32 16	1 26	251	91
d—*i*	8-9—8-9	32 59	1 41	241	123
d—*ϵ*	8-9—8-9	33 2	1 1	280	111
d—*y*	8-9—8	32 2	0 57	289	83
f—*g*	8-9—8	33 40	2 6	229	35
f—*h*	8-9—8-9	33 57	1 47	292	38
f—*i*	8-9—8-9	34 40	2 2	253	59
f—*k*	8-9—8-9	35 50	2 10	251	97
f—*ϵ*	8-9—8-9	34 43	1 23	317	85
f—*y*	8-9—8	33 43	1 19	338	76
g—*h*	8—8-9	34 51	1 59	346	38
g—*i*	8—8-9	35 33	2 14	281	31
g—*k*	8—8-9	36 44	2 21	263	66
g—*ϵ*	8—8-9	35 37	1 35	339	91
g—*y*	8—8	34 36	1 31	359	93
e—*f*	9—8-9	31 42	2 7	309	41
e—*g*	9—8	32 36	2 18	273	59
e—*h*	9—8-9	32 53	2 0	301	79
e—*i*	9—8-9	33 36	2 16	276	89
e—*y*	9—8	4 32 38	+1 32	328	113

Stars.	Mag.	A.R. of Middle Point. h m s	Dec. of Middle Point. ° ′	Position Angle. °	Dist.
y—*h*	8—8–9	4 34 54	+1 12	187	56
y—*i*	8—8–9	35 36	1 28	198	92
y—*k*	8—8–9	36 47	1 35	184	119
y—ϵ	8—8–9	35 39	0 48	255	31
h—*i*	8–9—8–9	35 51	1 56	215	37
h—*k*	8–9—8–9	37 1	2 3	231	72
h—ϵ	8–9—8–9	35 54	1 16	334	53
i—*k*	8–9—8–9	37 44	2 18	248	38
i—*l*	8–9—8	39 51	2 6	276	100
i—ϵ	8–9—8–9	36 37	1 32	359	79
ϵ—*k*	8–9—8–9	37 47	1 39	200	99
ϵ—*l*	8–9—8	39 55	1 26	235	119
ϵ—*m*	8–9—7	40 33	0 54	268	117
k—*l*	8–9—8	41 2	2 12	291	68
k—*m*	8–9—7	41 40	1 41	317	122
k—*x*	8–9—8	42 23	2 2	294	114
l—*m*	8—7	43 48	1 28	343	67
l—*n*	8—8–9	46 28	2 30	239	115
l—*o*	8—8	46 29	1 56	275	100
l—*x*	8—8	44 30	1 49	299	46
m—*o*	7—8	47 7	1 24	236	98
m—*x*	7—8	45 9	1 17	207	48
x—*n*	8—8–9	47 48	2 18	216	100
x—*o*	8—8	47 50	1 45	258	61
o—*r*	8—8–9	51 44	2 29	218	95
o—*s*	8—8–9	52 56	1 52	269	94
n—*o*	8–9—8	49 47	2 25	0	68
n—*r*	8–9—8–9	51 43	3 3	263	59
n—*s*	8–9—8–9	52 55	2 26	305	116
r—*s*	8–9—8–9	54 51	2 29	334	82
r—*t*	8–9—8–9	56 7	2 48	297	83
s—*t*	8–9—8–9	4 57 19	+2 11	226	53

Plan of Observation of Melpomene and Comparison Stars.

[P denotes the planet; other letters the stars in the preceding list.]

Date.	Star of Comparison.	Mag.	Hour Angle -4^h (Evening Observations). R.A. of Middle Point. h m s	Dec. of Middle Point. ° ′	Position Angle. °	Distance. ′	Hour Angle $+4^h$ (Morning Observations). R.A. of Middle Point. h m s	Dec. of Middle Point. ° ′	Position Angle. °	Distance. ′
Nov. 12	P—*t*	8-9	4 57 19	+2 28	267·9	38	4 57 11	+2 27	266·2	42
	P—*n*	8-9	52 54	2 43	108·4	99	52 47	2 42	109·9	96
	P—*o*	8	52 56	2 9	68·7	101	52 48	2 8	68·6	97
13	P—*t*	8-9	56 55	2 26	263·3	50	56 48	2 25	262·3	54
	P—*n*	8-9	52 31	2 41	113·5	90	52 24	2 40	115·3	87
	P—*o*	8	52 32	2 7	68·6	88	52 25	2 6	68·5	84
14	P—*t*	8-9	56 31	2 24	260·7	63	56 24	2 23	260·1	66
	P—*n*	8-9	52 7	2 39	119·6	81	52 0	2 38	121·8	79
	P—*o*	8	52 8	2 5	68·3	75	52 0	2 4	68·1	71
15	P *t*	8-9	56 7	2 22	259·3	76	55 59	2 21	258·9	79
	P—*r*	8-9	53 39	2 40	180·3	51	53 31	2 39	183·8	53
	P—*o*	8	51 43	2 3	67·4	62	51 36	2 2	67·1	58
16	P—*t*	8-9	55 41	2 20	258·5	89	55 32	2 19	258·4	93
	P—*r*	8-9	53 13	2 39	193·4	56	53 4	2 38	197·2	59
	P—*o*	8	51 18	2 1	65·6	49	51 9	2 0	64·6	45
17	P—*x*	8	4 48 53	+1 53	71·9	96	4 48 44	+1 52	71·6	91

Date.	Star of Comparison	Mag.	Hour Angle -4^h (Evening Observations). R.A. of Middle Point. h m s	Dec. of Middle Point. ° ′	Position Angle. °	Distance. ″	Hour Angle $+4^h$ (Morning Observations). R.A. of Middle Point. h m s	Dec. of Middle Point. ° ′	Position Angle. °	Distance. ″
Nov. 17	P—*r*	8–9	4 52 47	+2 37	204·1	64	4 52 38	+2 36	207·3	67
	P—*s*	8–9	53 59	2 0	284·3	64	53 50	1 59	282 6	68
18	P—*x*	8	48 26	1 51	71·1	82	48 17	1 51	70·7	77
	P—*r*	8–9	52 20	2 36	212·8	73	52 11	2 36	215.2	76
	P—*s*	8–9	53 32	1 58	279·5	77	53 23	1 58	278·3	81
19	P—*l*	8	46 37	2 1	89·1	104	46 28	2 1	89·6	99
	P—*s*	8–9	53 4	1 57	276·1	90	52 55	1 57	275·3	94
	P—*o*	8	49 57	1 56	22·1	11	49 48	1 56	358·2	10
20	P—*l*	8	46 9	2 0	90·7	90	46 0	1 59	91·2	85
	P—*s*	8–9	52 36	1 55	273·8	104	52 26	1 54	273·8	108
	P—*o*	8	49 28	1 55	308·9	13	49 19	1 54	296·4	16
21	P—*l*	8	45 41	1 58	92·6	76	45 31	1 58	93·3	71
	P—*n*	8–9	48 59	2 28	200·6	67	48 50	2 28	204·2	69
	P—*o*	8	49 0	1 54	283·1	25	48 50	1 54	279·7	29
22	P—*l*	8	45 12	1 57	95·1	62	45 2	1 57	96·2	57
	P—*n*	8–9	48 30	2 27	210·5	75	48 20	2 27	213·5	78
	P—*o*	8	48 31	1 53	275·2	39	48 21	1 53	273·8	44
23	P—*x*	8	46 3	1 45	22·0	16	45 53	1 45	4·0	14
	P—*l*	8	4 44 42	+1 56	98·9	47	4 44 32	+1 56	100·5	42

	P—*o*	8	4	48	1	+1	52	271·8	53	4	47	52	+1	52	271·3	58
24	P—*l*	8		44	12	1	56	105·6	33		44	3	1	56	109·0	28
	P—*m*	7		44	51	1	24	12·8	57		44	41	1	24	7·8	55
	P—*o*	8		47	32	1	51	270·1	68		47	22	1	51	269·9	73
25	P—*k*	8–9		41	35	2	7	113·6	88		41	25	2	7	115·1	83
	P—*m*	7		44	21	1	23	357·4	54		44	11	1	23	352·0	54
	P—*o*	8		47	2	1	52	269·2	83		46	52	1	52	269·1	88
26	P—*k*	8–9		41	5	2	7	118·8	75		40	55	2	7	120·9	70
	P—*m*	7		43	51	1	23	341·7	56		43	41	1	23	336·9	57
	P—*o*	8		46	32	1	53	268·8	98		46	22	1	53	268·8	103
27	P—*k*	8–9		40	35	2	7	126·0	62		40	25	2	7	129·0	58
	P—*l*	8		42	42	1	54	229·3	18		42	32	1	54	237·7	22
	P—*m*	7		43	20	1	22	328·0	62		43	10	1	22	324·1	65
28	P—*h*	8–9		38	11	1	44	84·9	92		38	1	1	44	84·6	87
	P—*l*	8		42	12	1	54	247·6	31		42	1	1	54	250·6	36
	P—*m*	7		42	50	1	22	317·4	71		42	40	1	22	314·5	74
29	P—*h*	8–9		37	41	1	44	84·0	77		37	31	1	44	83·5	72
	P—*k*	8–9		39	34	2	7	151·9	42		39	24	2	7	157·8	40
	P—*l*	8		41	41	1	54	254·8	46		41	31	1	54	256·4	50
30	P—*m*	7		41	49	1	22	303·7	94		41	38	1	22	302·2	98
	P—*l*	8		41	11	1	54	258·8	60		41	1	1	54	259·8	65
	P—*g*	8	4	36	53	+2	2	112·4	75	4	36	43	+2	2	113·7	71

Date.	Star of Comparison.	Mag.	Hour Angle -4^h (Evening Observations). R.A. of Middle Point. h m s	Dec. of Middle Point. ° ′	Position Angle. °	Distance. ″	Hour Angle $+4^h$ (Morning Observations). R.A. of Middle Point. h m s	Dec. of Middle Point. ° ′	Position Angle. °	Distance. ″
Dec. 1	P—*l*	8	4 40 40	+1 54	261·6	75	4 40 30	+1 54	262·3	80
	P—*f*	8–9	35 28	1 52	93·5	81	35 19	1 52	93·6	77
	P—*k*	8–9	38 32	2 7	196·7	38	38 23	2 7	203·4	39
2	P—*l*	8	40 10	1 55	263·6	90	40 0	1 55	264·1	95
	P—*f*	8–9	34 59	1 52	93·6	66	34 49	1 52	93·6	61
	P—*k*	8–9	38 2	2 7	216·3	44	37 52	2 7	221·5	46
3	P—*l*	8	39 39	1 56	265·2	105	39 29	1 56	265·6	110
	P—*f*	8–9	34 28	1 52	93·3	51	34 18	1 52	93·1	46
	P—*i*	8–9	36 22	2 0	196·3	21	36 11	2 0	209·0	22
4	P—*d*	8–9	31 12	1 33	70·7	126	31 2	1 33	69·7	122
	P—*f*	8–9	33 58	1 53	92·1	36	33 48	1 53	91·5	31
	P—*k*	8–9	37 2	2 9	240·0	65	36 52	2 9	242·5	69
5	P—*d*	8–9	30 43	1 32	67·4	113	30 33	1 32	66·1	109
	P—*f*	8–9	33 28	1 54	88·7	21	33 19	1 54	86·1	16
	P—*k*	8–9	36 32	2 10	246·7	77	36 22	2 10	248·5	81
6	P—*e*	9	31 55	2 8	121·2	45	31 45	2 8	124·0	40
	P—*f*	8–9	32 59	1 55	67·4	7	32 49	1 55	51·5	4
	P—*i*	8–9	34 53	2 4	254·1	52	34 43	2 5	256·2	57
7	P—*e*	9	4 31 26	+2 10	130·9	32	4 31 16	+2 11	136·5	28

	P—*f*	8-9	4	32	30	+1	56	303·0	10	4	32	20	+1	57	294·8	14
	P *i*	8-9		34	23	2	6	259·7	66		34	13	2	7	261·1	71
8	P—*e*	9		30	57	2	11	152·3	20		30	48	2	12	164·3	18
	P—*f*	8-9		32	1	1	57	289·4	24		31	51	1	58	298·1	29
	P—*d*	8-9		29	15	1	36	49·9	79		29	6	1	37	47·1	76
9	P—*c*	9		28	49	2	38	145·4	80		28	40	2	39	147·9	77
	P—*g*	8		32	27	2	11	259·3	64		32	17	2	12	260·8	69
	P—*d*	8 9		28	47	1	38	40·7	71		28	38	1	39	37·2	69
10	P—*b*	8-9		25	55	2	7	88·7	104		25	46	2	8	88·0	100
	P—*g*	8		31	59	2	12	263·7	78		31	50	2	13	264·8	82
	P—*d*	8-9		28	19	1	40	29·5	66		28	10	1	41	25·6	65
11	P *b*	8-9		25	28	2	9	86·2	91		25	19	2	10	85·2	87
	P—*g*	8		31	31	2	14	266·9	91		31	23	2	15	267·7	95
	P—*d*	8-9		27	52	1	41	17·3	64		27	43	1	42	13·1	64
12	P—*b*	8-9		25	1	2	11	82·6	78		24	53	2	12	81·2	74
	P—*g*	8		31	5	2	16	269·5	104		30	56	2	17	270·2	108
	P—*d*	8-9		27	25	1	44	4·8	65		27	16	1	45	1·1	66
13	P—*b*	8-9		24	35	2	13	77·6	66		24	26	2	14	75·5	62
	P—*c*	9		27	1	2	45	189·5	52		26	53	2	46	194·5	51
	P *e*	9		28	41	2	20	270·2	58		28	33	2	21	270·4	63
14	P—*b*	8-9		24	10	2	15	70·1	55		24	2	2	16	67·2	52
	P—*c*	9	4	26	36	+2	48	204·7	51	4	26	27	+2	49	209·4	52

Date.	Star of Comparison.	Mag.	Hour Angle -4^h (Evening Observations). R.A. of Middle Point. h m s	Dec. of Middle Point. ° ′	Position Angle. °	Distance. ′	Hour Angle $+4^h$ (Morning Observations). R.A. of Middle Point. h m s	Dec. of Middle Point. ′ °	Position Angle. °	Distance. ′
	P—*e*	9	4 28 16	+2 22	273·8	71	4 28 7	+2 23	274·6	75
Dec. 15	P—*β*	8	21 33	2 21	80·7	107	21 25	2 22	79·5	104
	P—*c*	9	26 11	2 50	218·8	53	26 4	2 51	223·2	55
	P—*e*	9	27 51	2 25	276·4	84	27 44	2 26	277·2	88
16	P—*β*	8	21 9	2 23	76·6	96	21 1	2 24	75·1	93
	P—*c*	9	25 47	2 52	231·2	58	25 39	2 53	234·6	60
	P—*e*	9	27 27	2 27	278·6	96	27 20	2 28	279·2	100
17	P—*a*	8	20 23	2 42	93·3	94	20 16	2 43	92.3	90
	P—*b*	8–9	22 58	2 23	25·4	37	22 51	2 24	19·2	37
	P—*c*	9	25 24	2 55	241·2	65	25 17	2 56	244·0	68
18	P—*a*	8	20 1	2 45	89·9	82	19 54	2 46	88·5	79
	P—*b*	8–9	22 36	2 26	7·3	40	22 29	2 27	2·1	40
	P—*c*	9	4 25 2	+2 58	249·2	73	4 24 55	+2 59	251·5	75

The Approaching Opposition of Mars. By R. A. Proctor, Esq.

Want of leisure has prevented me from preparing, as I had intended, a paper indicating the probable appearance of *Mars*, on different days, during his approaching opposition. Herr Marth's excellent Ephemeris in the last number of the *Notices* in large degree subserves the purpose of anticipatory charts, such as I drew in March 1873; for it is easy from the data he supplies to make orthogonal projections for any given day and hour.

My chief object in writing now, however, is to express my agreement with Dr. Terby, of Louvain, as to the doubtful features of my chart. Indeed several of the doubts he suggests had been indicated by the use of dotted outlines in my map. I agree with him also in considering that M. Flammarion's course, in *making* all the corrections which Dr. Terby says rightly may *have* to be made, is too summary. In some respects, indeed, the corrected chart is certainly incorrect. Thus, it carries Nasmyth Inlet eastwards as a narrow strait to join Tycho Sea, which may or may not be right; but a correction which will certainly have to be made in this part of the map is overlooked altogether. I refer to a bay running southwards from this strait into Dawes Continent. This is shown well in drawings by De la Rue, Knobel, and Green. I find it is shown in several of the views given me by Dawes, though not in the best. I ought to have remembered that positive evidence for a dark marking is altogether stronger than negative evidence against it. For a similar reason, on the other hand, I think it unlikely that the J. Herschel Strait of my chart does not extend into De la Rue Ocean. For some of Dawes's best drawings, in making which he paid special attention to this region, show the strait continuous. See especially his drawing numbered 10 in vol. xxv. of our *Notices*. See also Mr. Knobel's excellent series of drawings in vol. xxxiii. This positive evidence seems conclusive. The marked gap in Lockyer's views (among the best we have) indicates that clouds had gathered over that region—where the strait is manifestly shallow, so that mid-day clouds would readily form, as over shallow seas on our own Earth.

I have here used my own nomenclature, which was devised simply for convenience of reference. If Dr. Terby had altered the names, all or any, I should have adopted the names he used. But as he, who has given so much attention to the subject and studied so many drawings, has adopted my nomenclature, I have not thought it necessary to follow M. Flammarion (who employs Dr. Terby's results and my own) in changing most of the names. The changes seem uncalled for and likely to introduce confusion.

2 *North Road, Clapham Park, S.W.*

Note on the Coming Opposition of Mars. By N. E. Green, Esq.

In order to utilise the Opposition of this interesting planet, and secure a fair amount of uniformity in the drawings of various observers, special forms have been printed on a paper possessing peculiar facilities for the production of effect.

This paper consists of an under-surface of white enamel covered with a warm tone closely resembling the general tint of *Mars*. Upon this surface the dark marks may be drawn either with chalk or pencil, and softened with the stump in the usual manner.

The peculiar advantage consists in the use of the knife, by which the most delicate alterations of tint or form may be effected, or the warm tone partially removed for faint lights, or entirely erased for the polar snows.

The only care required is to avoid the use of india-rubber, all general softening or removal being effected by bread.

Sheets of these forms may be obtained at the monthly meeting in June, or by letter to N. E. Green, 3 Circus Road, St. John's Wood, who will feel much pleasure in supplying them to the Fellows.

Note on the Distribution of the Fixed Stars.
By R. A. Proctor, Esq.

Von Littrow has shown that when only the numerical relations of stars of various orders of brightness are considered, the increase in the number of the fainter stars corresponds with the theory that apparent magnitude depends chiefly on distance. When, however, the localisation of stars of various orders is considered, we find evidence of the numerical superabundance of stars really smaller than their neighbours. To take a rough illustration of the effect of this consideration: If, in examining any telescopic field taken at random, we find for each increase of illuminating power an increasing number of stars, the inference is that we are penetrating further and further into space. But this inference must be rejected if we find that the field of view lies in either of the *nubeculæ*, and that the increase is not recognised, or only occurs in a markedly less degree, outside the borders of the star-cloud. In like manner, though not so obviously to ordinary vision, we find evidence of the existence of clouds of stars or streams of stars (such as are found in various parts of the Milky Way); and numerical increase with increasing powers, if found within such clouds and in markedly less degree outside their borders, cannot be referred to increase of distance, but, according to the laws of probability, must be explained as due to the recognition of stars really smaller lying within the limits

of such clouds or streams. (By streams I do not mean rows of stars, but stream-shaped *regions* in which are many thousands of stars.) I think my Chart of 324,000 Stars in the Northern Hemisphere demonstrates this, especially when it is remembered that the regions where *these* stars (largely exceeding in number those dealt with statistically by Von Littrow, or indeed, so far as I know, by any other) chiefly congregate, are precisely the regions where stars down to the 20th magnitude are found in greatest number. As Mr. W. M. Christie remarks in a very able and impartial review of my researches (*Academy* for June 27, 1874), "it is hardly conceivable that" the clustering aggregations "should be enormously long cylinders or spindles, turned in every case exactly towards us."

It appears to me that direct evidence based on the laws of local aggregation, as deduced from isographic charting (which is in effect statistical enumeration recorded so that all its classifications can be *seen*), cannot be disposed of by a less perfect method of counting, in which averages only are considered without reference to localisation. This would be much as though an archæologist's theory respecting certain mounds in a plain should be met by a surveyor with the argument that the mounds had no real existence, seeing that, if they were levelled, the plain would not rise appreciably, where they had been, above the surrounding level. If the mounds could be thus disposed of, any theory respecting them would fall to the ground. In like manner, I must admit that if no attention be paid to the evidence of stellar aggregation in particular regions of the heavens, there remains little evidence of the existence of real stellar aggregations in space.

Visible Transits of Mercury to the Year 2000.

By Rev. S. J. Johnson.

Upon recently making an examination, for curiosity, of the transits of *Mercury* that were to be expected in this land till the close of the next century, I found the method of cycles, even that of 263 years, would not answer in many instances, as, owing to the difference of the planet's geocentric latitude at the return of the period, the duration of the transit may vary considerably. I therefore computed the places of ☉ and ☿ from some approximate tables—those given in Lindsay's *Chrono-Astrolabe*—and the results will, I believe, deviate little from the truth.

Thirteen transits of *Mercury* fall in the present century, but one only, that of 1832, was visible throughout at Greenwich. In the next century twelve happen, four of them being entirely visible there.

After the transit of 1878 there will be no opportunity for a good observation of this sort in the remaining years of this

century, as in 1891 the planet leaves the Sun scarcely half an hour after sunrise, and in 1894 the ingress is but a quarter of an hour before sunset.

Sometimes a period of thirty years elapses that is transitless so far as we are concerned in this country, as between the years 1802 and 1832, and from 1973 to 2003.

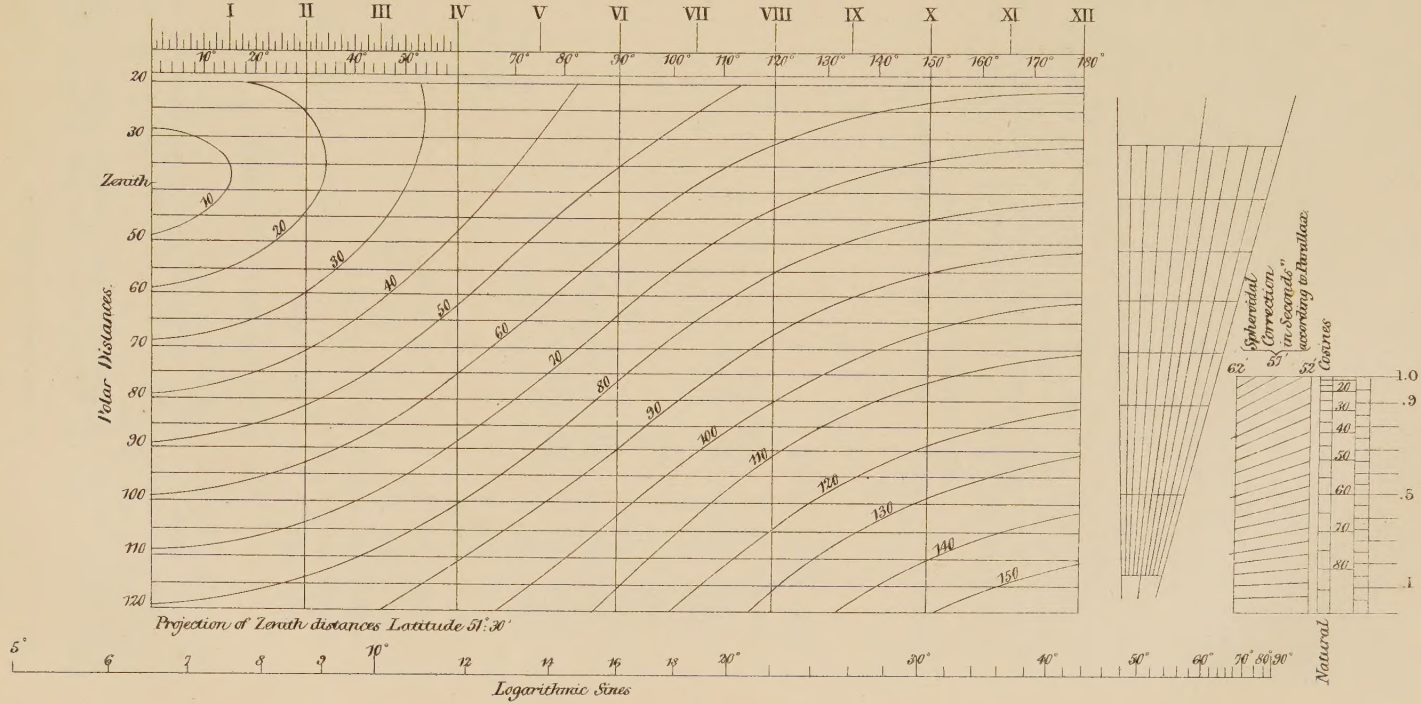
(Contacts below the horizon are omitted.)

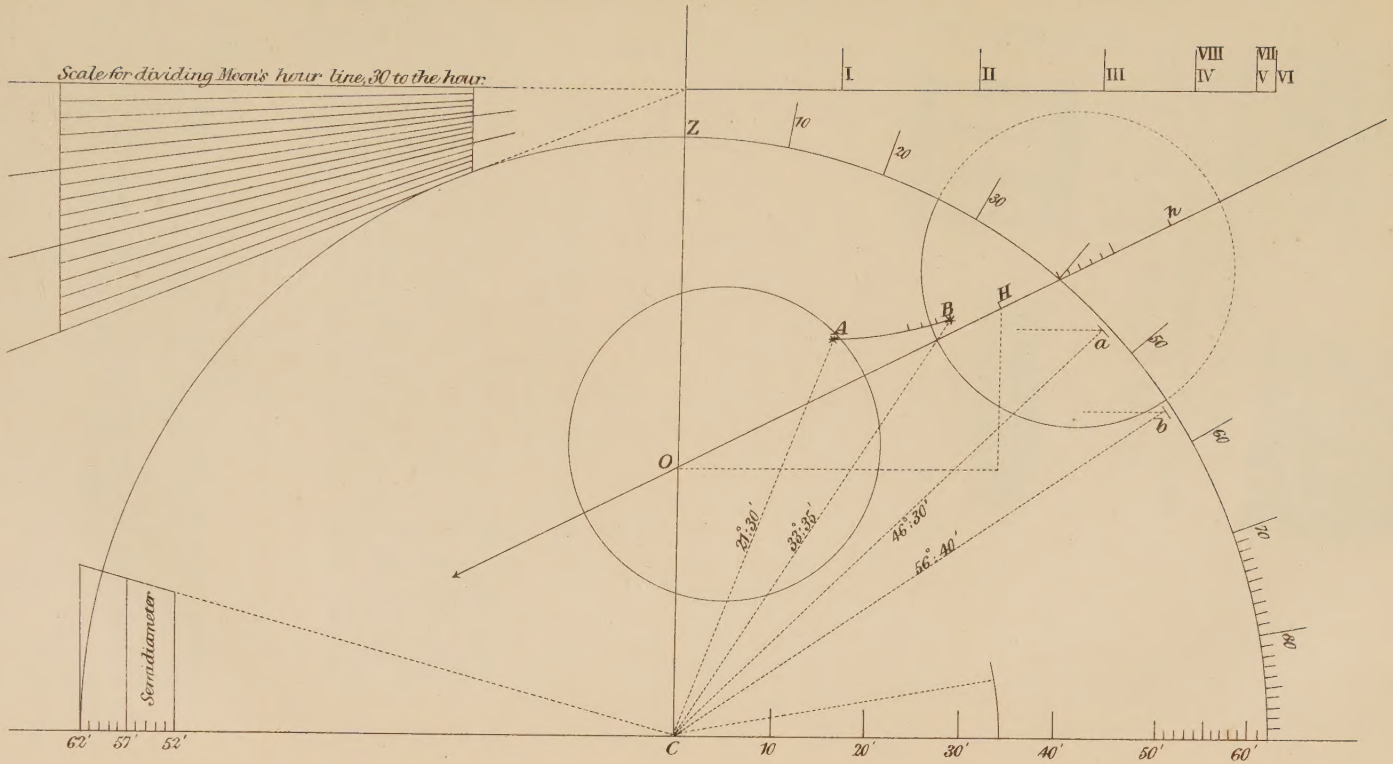
1881	Nov. 7	Below the horizon.			
1891	May 10	„ „		Egress about $4\frac{3}{4}^h$ morn.	
1894	Nov. 10	Ingress about 4^h aft.			
1907	Nov. 12	„ „	$10\frac{1}{4}^h$ morn.	„ „	$1\frac{3}{4}^h$ aft.
1914	Nov. 6	„ „	$9\frac{3}{4}^h$ morn.	„ „	2^h aft.
1924	May 7			„ „	$5\frac{1}{2}^h$ morn.
1927	Nov. 8			„ „	$8\frac{1}{4}^h$ morn.
1937	May 10	„ „	8^h morn.	„ „	$9\frac{1}{4}^h$ morn.
1940	Nov. 10	Below the horizon.			
1953	Nov. 13	Ingress about $3\frac{1}{4}^h$ aft.			
1960	Nov. 6	„ „	$2\frac{1}{4}^h$ aft.		
1970	May 9			„ „	noon.
1973	Nov. 9	„ „	$7\frac{3}{4}^h$ morn.	„ „	1^h aft.
1986	Nov. 12	Below horizon.			
1999	Nov. 14	Below horizon.			

Upton Helions Rectory,
Crediton.

Extract from a Letter received by Mr. Ranyard from Prof. Meucci of the Museo Reale, Florence.

I have recently found, in the collection of instruments under my care, an Arabian Celestial Globe, which I believe is certainly the oldest of the few in existence; its date is 1081 A.D. It is made of metal, and its diameter is 209 millimeters. It represents the celestial sphere as given by Ptolemy, except that the longitudes of all the stars are augmented by 14° 10′, so that we find *Regulus* at 16° 40′ ♌ &c. The names of the constellations are all in cufic characters. There is also an inscription in the same character to the effect that the globe was made by Ibrâhîm Ibn Saîd al Sahlî al Wazzon (the Weigher), with his son Muhammed. The work, it appears, was finished in Valencia, in the beginning of the month Safar, in the year of the Hegira 474 (this month began July 11, A.D. 1081), and it was made for Abu Isa Ibn Labbun (or Lebûn).


I
II
III
IV
V
VI
VII
VIII
IX
X
XI
XII
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°
120°
130°
140°
150°
160°
170°
180°
Zenith
Polar Distances.
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
Projection of Zenith distances Latitude 51°30′
Spherical Correction in Seconds according to Parallax
62
57
52
Cosines
1.0
.9
.5
.1
5°
6
7
8
9
10°
12
14
16
18
20°
30°
40°
50°
60°
70°
80°
90°
Logarithmic Sines
Natural


Scale for dividing Moon's hour line, 30 to the hour.
I
II
III
VIII
IV
VII
V
VI
Z
10
20
30
40
50
60
A
B
H
a
b
O
21°.30′
32°.35′
46°.30′
56°.40′
Semidiameter
62′
57′
52′
C
10
20′
30′
40′
50′
60′